One
Two

Series B

Rich Learning Tasks

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Problem Solving and Reasoning

One Two

Contents

Name:

Dogs and Cats

Number

There are some dogs and some cats. There are 4 more dogs than cats. There is an odd number of cats.

- How many animals could there be?
- How many animals could there not be?

Reasoning behind the task

This task enables students to practise addition in a real-life situation. Students can access this task at different levels. Some students will select only small numbers while others will work with larger numbers.

The task encourages students to observe patterns (e.g. 6, 10, 14, 18, ... total animals is possible but 5, 7, 9, 11, ... is not) and to reason about why the patterns occur. Some students may generalise that these patterns must continue.

Curriculum coverage

- Number
- Addition
- Pattern recognition

Expectations

All	Most	Some
• Calculate some numbers that work and at least one that does not work for the total number of animals.	• Calculate several numbers that work and several that do not work for the total number of animals. • Show why numbers below 6 for the total number of animals is impossible.	• Generalise that any odd number greater than or equal to 5 is possible for the number of dogs. • Describe the number of each type of animal when given the total number of animals.

Dogs and Cats

Key questions

- Could there be 2 dogs and 2 cats? Why not?
- If there were 3 cats, how many dogs would there be? How many animals?
- If there were 5 dogs, how many cats would there be? How many animals?
- Were there more dogs or cats? Why?
- What is the least number of animals there could have been?
- What total number of animals is impossible? Why?

Scaffolding learning

- Start off with a small number of cats, e.g. 1. How many dogs would there be? How many animals would there be?
- Repeat for different numbers of cats and dogs
- Why can there not be 4 cats?
- Look for patterns. What do you notice?
- Is there a maximum number of animals there could be?

Challenge

Change the rule to having 5 more dogs than cats to see how the possible and impossible numbers change. Use this task to explore subtraction by providing students with a total number of animals and asking them how many cats and dogs there could be.

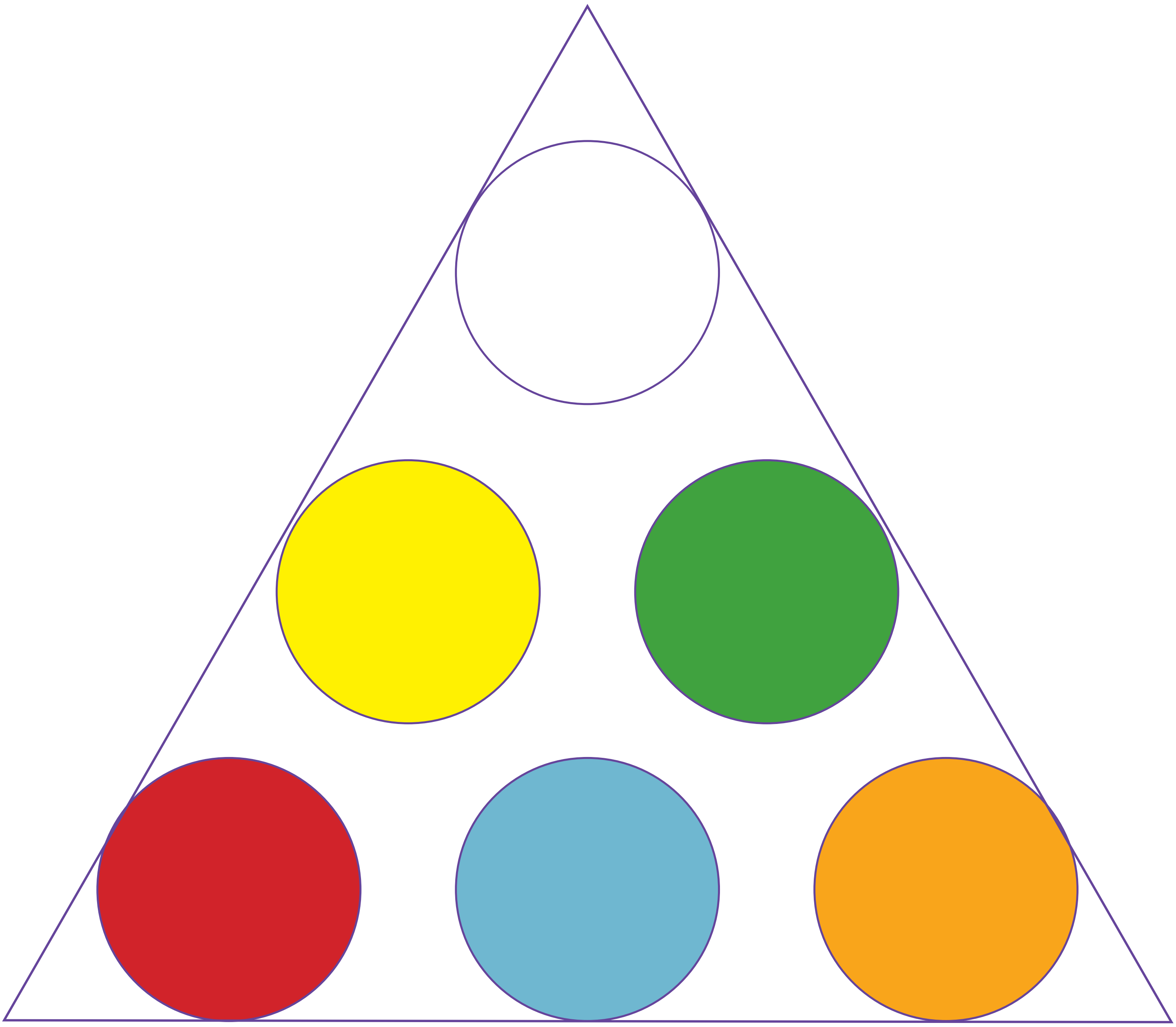
Name:

Six Circles

Number

Arrange dots in the circles so that there are the same total number of dots on each side of the triangle, but not the same number in every circle on that side.

- How many dots are there on each side of the triangle?
- Try to find several solutions.



Reasoning behind the task

The task is designed to provide students with an opportunity to practise simple addition, subtraction, and decomposition of numbers. For example, a student who uses 1 dot in the white circle, 2 dots in the yellow circle and 5 dots in the red circle can either investigate other decompositions of 8 that involve 1 or 2 or 5, or can subtract one of those numbers from 8 to figure out what to put in other circles.

Some students will stop with one possibility, but they should be encouraged to look for others. Many students will not realise that once a particular solution works, the same number of dots could be added to each circle to generate another solution. Others will consider orientation and realise that rotating the triangle (e.g. putting the red circle at the top) yields what could be considered a different solution.

Curriculum coverage

- Number
- Addition
- Subtraction
- Decomposing numbers

Expectations

All	Most	Some
• Create at least one solution that meets the criteria.	• Create several solutions using reasoning effectively.	• Create many appropriate solutions, using reasoning to generate at least one solution based on a previous solution.

Key questions

- Is it possible to have 1 dot in the white circle? How about 4 dots?
- Suppose you want each side to use a total of 6 dots. How could you arrange those dots into three groups?
- What happens if you put 3 dots in the white circle, 2 in the yellow and 4 in the red? How many dots could you put in the green and orange circles?
- Once you have worked out one possible layout of dots, how could you use this to help solve other possible layouts?

Scaffolding learning

- Start off with a small number of dots in each circle on one side of the triangle, e.g. 1, 2, 3 dots.
- Add them up and work out what dots you will need to add to the other circles.
- Change the dots around or change the amount to make all the sides add up to the same total.
- Explore different numbers of dots in each circle to solve the problem in different ways.

Challenge

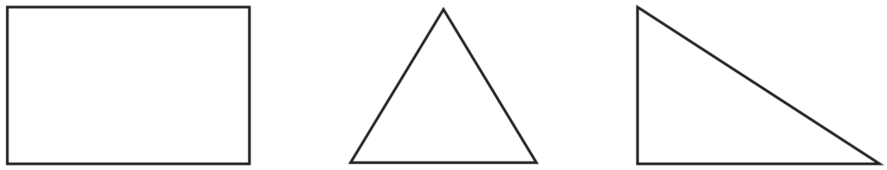
- Use exactly 24 dots in total.
- Make each side equal 10 dots.

Name:

Four Shapes

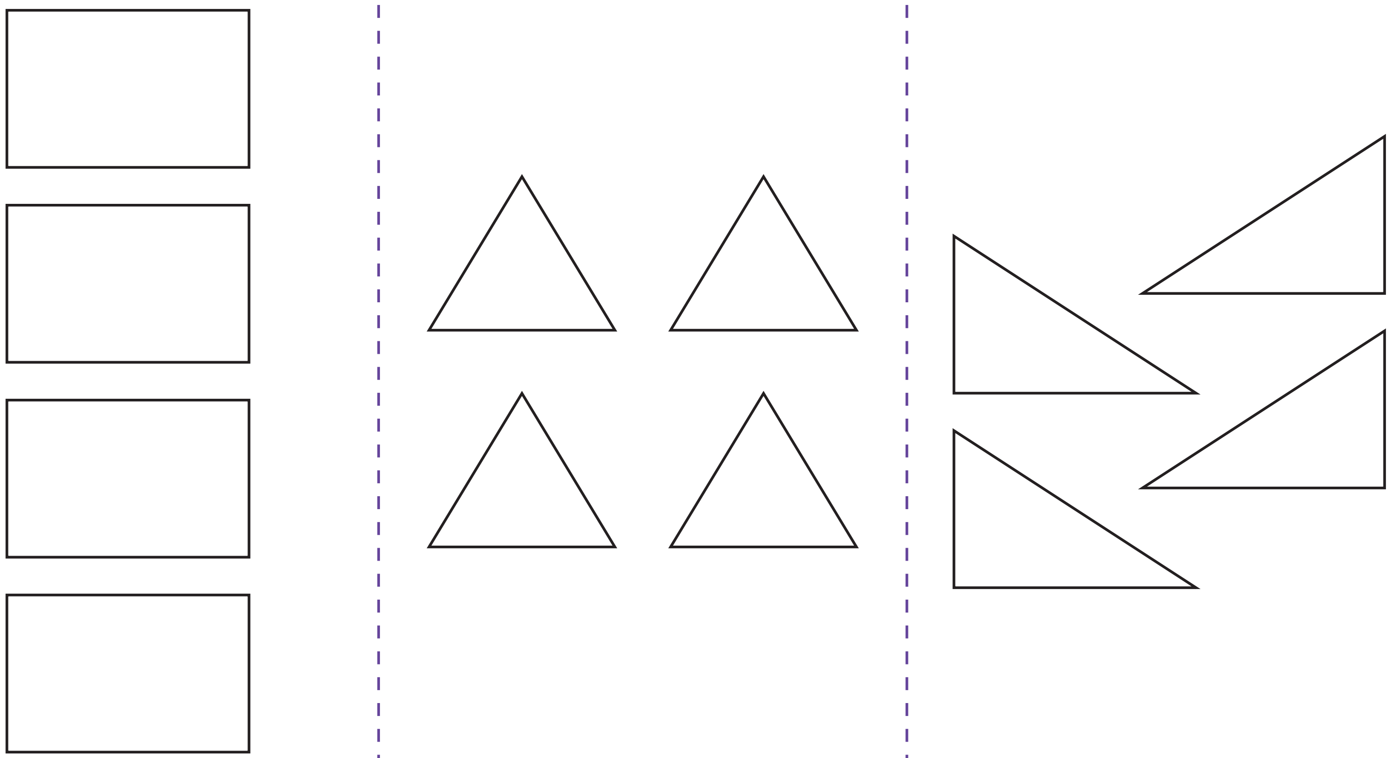
Geometry

Choose one of these three shapes.



Which of the shapes in the list below do you predict that you can make if you put together 4 copies of your shape?
Explain your prediction.

- a square
- a rectangle
- a triangle
- a 6-sided shape
- a 5-sided shape
- an 8-sided shape
- a circle



Four Shapes

Reasoning behind the task

Students practise composing bigger shapes from smaller ones using visualisation and reasoning skills. Students might reason that it is impossible to create a triangle using rectangles, but that they might be able to create a rectangle using triangles; that if they use two rectangles to make a square, they won't be able to use four; that four smaller equilateral triangles can make a bigger one.

Curriculum coverage

- Geometry
- Classifying shapes
- Creating patterns with shapes
- Creating shape

Expectations

All	Most	Some
• Predict at least one possible shape and model it using the four copies.	• Predict several possible shapes, explaining their reasoning and modelling them accurately.	• Correctly predict which of the shapes are possible with good explanations of why. • Arrange and rearrange shapes using good problem solving skills to create new shapes with the 4 copies.

Key questions

- Which shape was the easiest to make? Why?
- Which of your predictions didn't work?
- What shape did you use to make a big rectangle? How did you connect those four shapes?
- Were you able to make a triangle?
- Could you make a triangle using rectangles?
- Why couldn't you make a circle?

Scaffolding learning

- Cut out the shapes and explore fitting them together in different ways
- Try putting the rectangles on top of each other
- Try turning the shapes around
- Count the number of sides your new shape has – what happens to the number of sides if you slide one of the shapes across a bit?

Challenge

What is the minimum and maximum number of sides you can create when making a new large shape? Try this for each type of original shape.

Name:

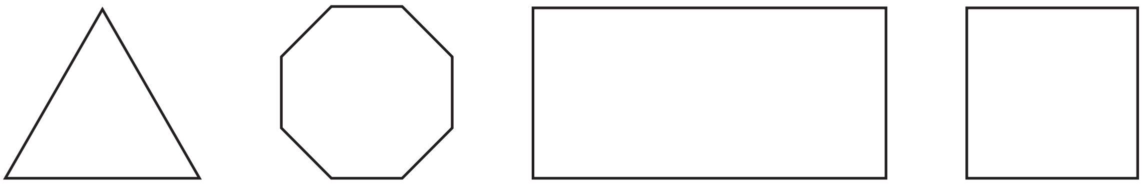
Geometry

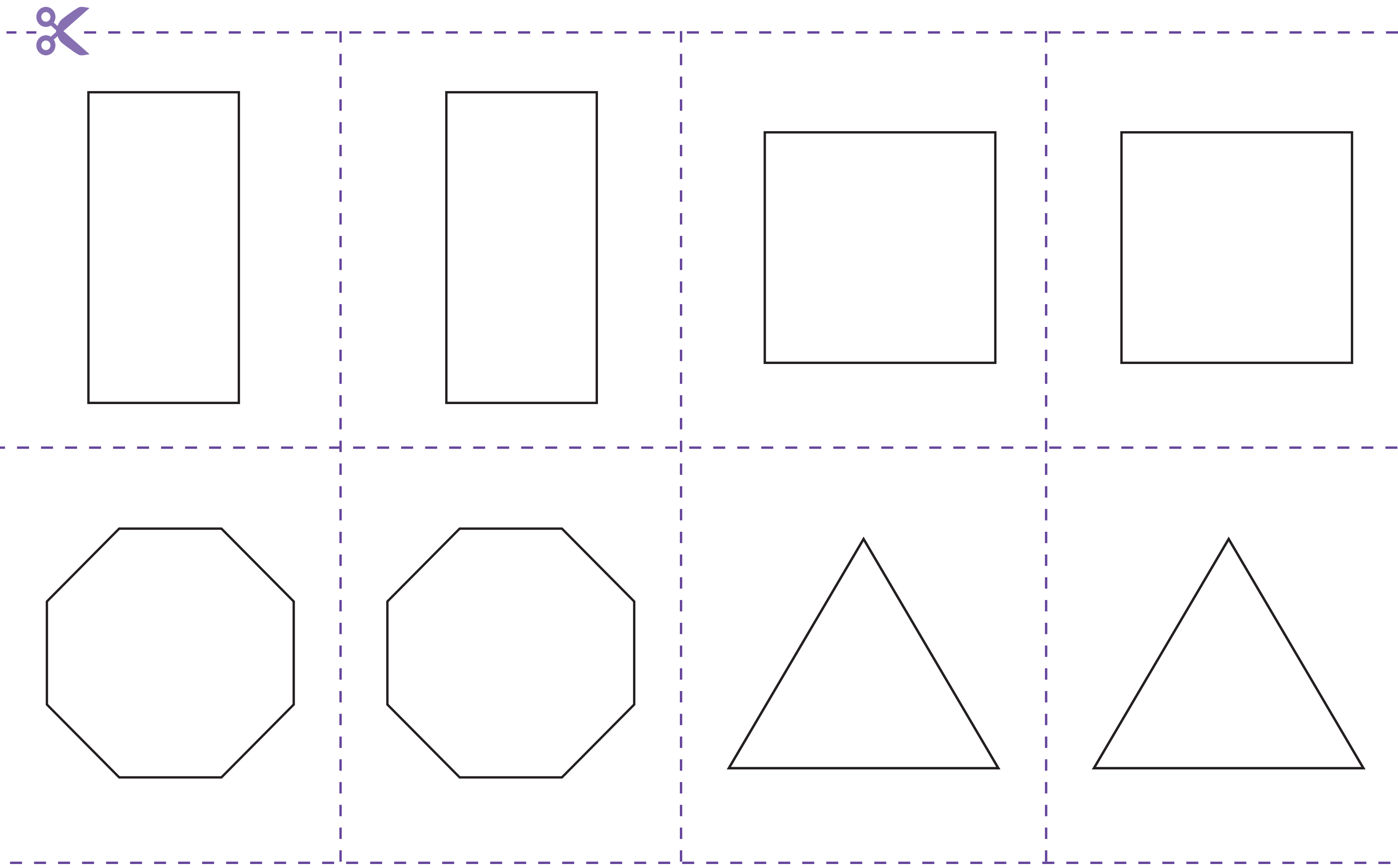
Share It

Choose two of these shapes.

Cut the shape into 4 equal parts in as many ways as you can.

How do you know that the parts are equal?





Reasoning behind the task

This task focuses students on how some shapes can be partitioned to create other shapes. At the same time, it introduces students to the fractional concept of quarters, or fourths. By allowing students the choice of shapes, differentiation of learning is addressed.

Each of the shapes provided has symmetry to allow for the first cut into halves to be relatively easy. Asking children to cut up the shapes in as many ways as they can encourages flexibility of thinking.

It may be necessary to point out that equal means equal in area (that if they were pieces of cake, all of them would be fair), not just equal in number.

Curriculum coverage

- Geometry
- Dividing into equal parts
- Fractions (quarters)

Expectations

All	Most	Some
• Partition the rectangle and/or square into four equal parts at least one way.	• Partition the rectangle and square into four equal parts in at least two different ways. • Provide some reasoning as to why each part is equal.	• Partition three or more of the shapes into four equal parts in at least two different ways. • Explain clearly using reasoning how the partitioned the shape and how they know the sections are equal.

Key questions

- Do you predict that your pieces will be the same shape as you started with? Why / why not?
- Could you use your understanding of symmetry to help you?
- Once you had created four equal pieces one way, did it help you to solve doing it another way?
- Did all four pieces have to look exactly the same to be equal? If not, what is equal about them?
- Did you cut straight across the shape? Did you have to do it like that?
- Which shape do you think might be easiest for you to partition? Why?
- Might it help to cut it into two shares first? How would that help?
- Suppose you used a line straight down to get you going. Where would you put that line?

Scaffolding learning

- Think about which shape is going to be the easiest to cut into halves and quarters
- Cut it into two equal pieces first
- Think about using a straight line to cut your shape – where will you put that line?
- Do you need to cut straight across? Can you cut it in a different way, still making sure your pieces are equal?

Challenge

- Cut the triangle into four equal pieces.
- Draw an irregular shape and consider how to partition that into four equal shapes.

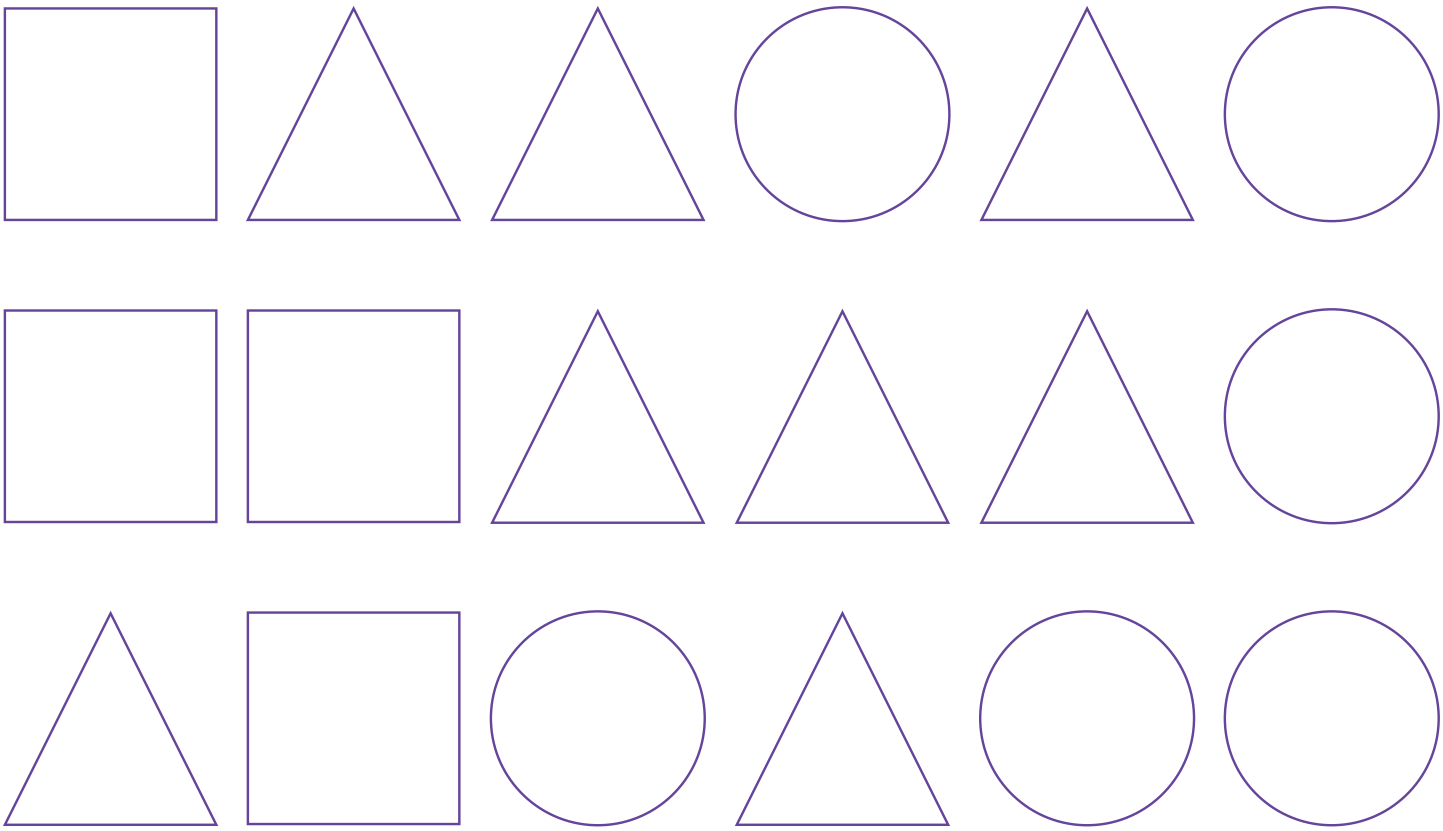
Name:

Yellow, Red and Blue Shapes

Colour the shapes below so that there are three times as many yellow ones than red ones, and twice as many blue ones than red ones.

How many yellow ones are coloured?

Make all the circles blue.



Yellow, Red and Blue Shapes

Reasoning behind the task

In this task, students are asked to solve a problem using multiplication or division, by calculating using an array of three different shapes. The task could be simplified by asking students to only follow one criteria e.g. colour twice as many blue ones than red ones. At first the type of shape is irrelevant, but students could be challenged to colour shapes to match the criteria and type of shape, e.g. colour all the circles blue, colour half the squares red etc.

Curriculum coverage

- Statistics and Data
- Sorting 3-D shapes
- Comparing numbers
- Partitioning numbers

Expectations

All	Most	Some
• Colour the shapes according to one or more of the colour criteria.	• Colour the shapes according to both colour criteria and is able to make all the circles blue still following the criteria.	• Follow the colour criteria and be able to describe how many e.g. red shapes there would be if there were 12 or 24 shapes in total, showing an understanding of the relationship between the number of each coloured shapes and the total amount.

Yellow, Red and Blue Shapes

Key questions

- If you have 2 red shapes, how many yellow ones and blue ones will you need?
- Can you make all the squares red? Why / why not?
- Can you make all triangles yellow? Why / why not?
- How many shapes are there altogether? Does this help solve the problem?
- How many red shapes would you have if there were only 12 shapes in total? What about 24 shapes in total

Scaffolding learning

- Start off with colouring 1 red shape and explore colouring the yellow and blue shapes following the criteria of 3 times more yellow and double the number of blue.
- How many shapes are left? How many more red ones can you colour?
- Colour all the shapes in – how many are red, yellow and blue.

Challenge

- Follow the criteria for 24 shapes.
- Without drawing them all, how many yellow shapes would there be if there were 20 red shapes? How many circles would be needed?
- Add more colours and introduce your own criteria for a partner to follow.

Name:

Pizza

Data

A pizza restaurant recorded the number of different toppings that were ordered on one evening.

Twice as many ordered pepperoni as extra cheese.

Half as many ordered mushrooms as extra cheese.

Record the number of orders that might fit in each category:

- pepperoni
- extra cheese
- mushrooms.

Record the total number of orders.

Explain your working out.

Come up with several possibilities.

Reasoning behind the task

This task requires students to sort objects into categories meeting given conditions. The references to **half** and **twice** informally serve as a prelude to fractional and proportional reasoning as well as to the inverse relationship between multiplying and dividing. (You might need to define **twice** and **half** to remind some students that twice means two of something and half means that it would take two to make a number.)

Because students can choose the numbers they work with, this task suits students at many levels. You may want to suggest that more able students use a graph to record their suggested results.

Curriculum coverage

- Statistics and Data
- Sorting objects by one or more criterion
- Fractional language
- Inverse relationships

Expectations

All	Most	Some
• Choose the correct numbers for one of the conditions in the problem, e.g. doubling OR halving for at least one extra cheese value.	• Choose the correct numbers to meet the conditions of the problem for several values and were able to explain the relationship between values. • Understand how to find the topping values once one value was selected.	• Choose the correct numbers to meet the conditions of the problem for a range of values and were able to explain why the numbers made sense. • Understand that once one topping value was selected the others were fixed. • Use the relationship between twice and half to explain that extra cheese was double the mushroom value and half the pepperoni value.

Key questions

- Which number did you decide on first? Was it easy to work out the other values?
- How did the numbers of those who ordered mushrooms and those who ordered extra cheese compare?
- Which was the most popular topping choice?
- Suppose two more people ordered mushrooms – what other pieces of information will you need to change? How would you change it?
- Can you work out how many people in total participated in your survey?

Scaffolding learning

- Choose a small number for the number of people who ordered extra cheese, e.g. 2 (don't choose 1 as you won't be able to halve it to find the number of mushroom orders).
- Double it to find the number who ordered pepperoni and halve it to find the mushroom orders
- Try out different numbers and look for patterns
- What is the total number of orders each time?

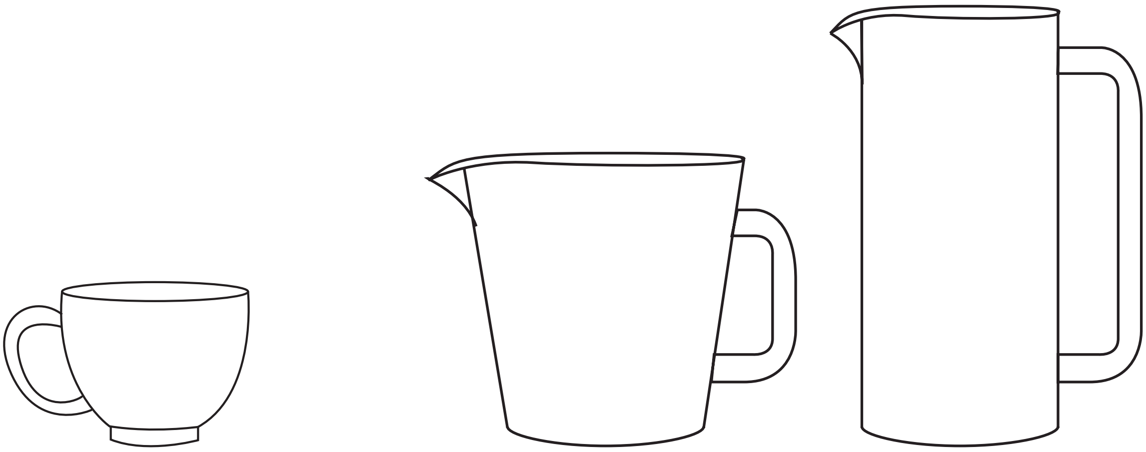
Challenge

- Create new relationships among the three toppings and choose numbers for those new relationships, e.g. it might be that three more people liked pepperoni than extra cheese or maybe twice as many liked mushrooms as pepperoni.
- Imagine that each person chose 2 toppings - work out the total number of people who ordered.

Name:

Container Conundrum

You will need these three containers:



Which container do you think will hold the most? Why?

Predict how full the largest container will be if you pour a full cup into it 2 times. Check your prediction.

Then predict for 3 times, 4 times, and 5 times. Check each time.

Explain how you used what you learnt about pouring 2 times to help you with your other predictions.

Container Conundrum

Reasoning behind the task

This task helps students consider units of capacity and fosters proportional reasoning by asking students to relate the sizes of two capacities. The initial question about which jug holds more is designed to help students realise they have to look at more than one attribute when considering volume/capacity: it is about both height and width. Emphasise this by widening the flare of each container.

Asking students to make predictions will help with their future capacity reasoning skills.

Curriculum coverage

- Measurement
- Volume and capacity
- Estimation

Expectations

All	Most	Some
• Predict what might happen, and explore how many cupfuls of water will fit into the two jugs, describing what they see.	• Make reasonable predictions most of the time, using what they see to help them amend or make future predictions.	• Make consistent and reasonable, well thought-out predictions, explaining how earlier predictions and outcomes informed later ones. • Aware that short, wide containers could hold the same or more than tall, narrow containers.

Container Conundrum

Key questions

- Do you think the smaller of the two jugs will hold more than 3 cups? Why / why not?
- Which jug will hold the most water?
- Could the jugs hold the same amount even though they are not the same height? Why / why not?
- Was it easier to predict how many full cups would fit in the tall, straight container or the container which got wider at the top? Why?
- Once you knew how high the water went when 2 cups were poured into the jug, was it easy to predict for 3 cups? 4 cups? Why?
- Did your predictions improve?

Scaffolding learning

- Pour one full cup into one of the jugs and watch what happens.
- Predict what will happen if you pour another full cup into it. How many cupfuls do you think the container will hold?
- Explore pouring water into both jugs – which one do you think will hold the most water?

Challenge

Explore a wider range of containers and make predictions and comparisons.

Name:

Placemats

Which placemat do you think takes up more space? Why?

Make a rectangular placemat that would take up more space than the black one, but not as much space as two of them.

How do you know you're right?

Placemats

Reasoning behind the task

In this task students are required to compare areas. Students may superimpose one placemat on top of the other to make a direct comparison, but will have to do some cutting and rearranging to be certain which shape has the greater area.

Some students may choose to unitise both areas by putting square or rectangular units on top of both shapes.

To make their own placemats, students might copy one of the placemats twice and cut some off or very slightly enlarge one or both dimensions of one of the existing placemats. There could be discussions among students about which way they would find easiest.

Curriculum coverage

- Measurement
- Comparing area

Expectations

All	Most	Some
• Recognise that the grey placemat has more area. • Make a placemat of the appropriate size.	• Recognise that the grey placemat has more area and attempts to prove it by either unitising or cutting up and rearranging, but does not address imprecisions (e.g. leaving gaps between units or not quite covering a piece). • Make a placemat of the appropriate size and begins to explain why they are correct.	• Recognise that the grey placemat has a greater area and proves it by unitising with shapes or by cutting up and rearranging pieces from one placemat on top of the other. • Make a placemat of the appropriate size and clearly explains why they are correct.

Placemats

Key questions

- Was it easy to tell right away which placemat takes up more space? Why / why not?
- How sure are you about which placemat takes up more space? What did you have to do to check?
- Which placemat is wider? Which is higher? Why do you need to think about both of those things?
- Suppose you move the black placemat on top of the grey one. How could this help you solve the problem? Would cutting one of the placemats help?
- To make your new placemat, did you have to change the width and height of your original placemat?

Scaffolding learning

- Think about the width and height of both placemats.
- Try putting one of the placemats on top of the other – cut off the overlapping parts and lay on the top.
- Think about how to make a placemat that will take up more space that the black one: try copying the original and cutting a bit off, or slightly enlarge both dimensions.

Challenge

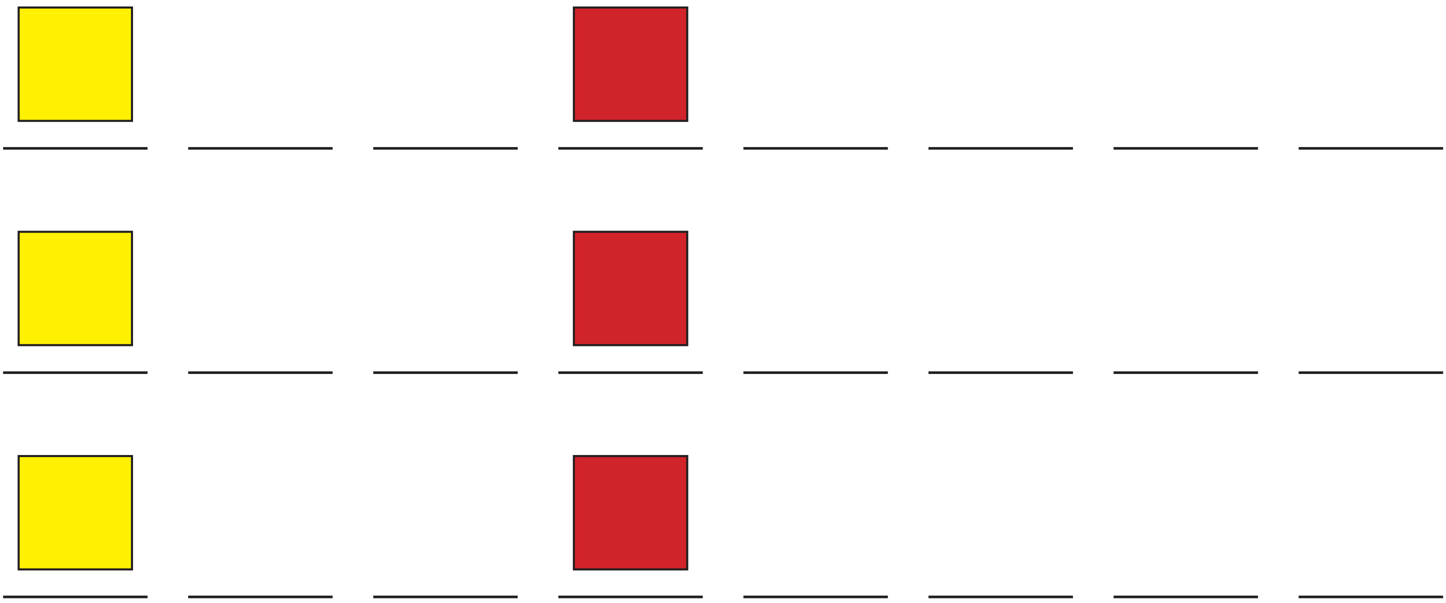
Create placemats to meet other conditions, e.g. just as big as the black one, but wider, or as big as more than two but less than three of the black one.

Name:

Fill Me In

This is part of a pattern. What could the rest of the shapes in the pattern look like?

Try several times, making different patterns each time.



Reasoning behind the task

This task helps students realise that a few terms in a pattern may not show the pattern; there are, in fact, an infinite number of possible patterns containing any small set of terms. Rather than starting with the yellow and red squares and asking students to continue the pattern in different ways, the two given terms are spaced apart; it is usually trickier for students when any of the first few terms are missing, even though it is really no different to the first two terms being given.

Both shapes given were squares, and so some students will assume that all the shapes must be squares, even though that is not the case. Similarly, yellow and red were used, so some students will assume that all shapes must be yellow and red, whereas others will realise other colours can be introduced.

Curriculum coverage

- Pattern and relationships

Expectations

All	Most	Some
• Create at least one correct pattern describing why it is a pattern.	• Create several correct patterns explaining why they are patterns.	• Create several correct patterns justifying why they are patterns. • Use one pattern to suggest another, explaining why certain conditions are / are not possible in creating patterns.

Key questions

- Do all of the shapes have to be squares? Could they be?
- What colours did you use in your patterns? Could there have been other colours?
- Could the 2nd and 3rd shapes be completely different from the 1st and 4th?
- Once you had one pattern, did it help you work out others?
- What made your patterns correct? What makes them patterns?
- How will you make sure that what you create really is a pattern?

Scaffolding learning

- Think about what shapes and colours you will use.
- Start off with a simple repeating pattern.
- Make your pattern more complicated using different shapes and colours.

Challenge

Create patterns that meet certain conditions, e.g. there are a lot more yellow shapes than red ones, OR, there are at least three different shapes in the pattern, OR the 5th and 7th shapes have to be the same.

Name:

Make It Balance

You will need:

- a balance scale
- red, yellow, blue, green and brown cubes.

One side of a balance has red and yellow cubes.
The other side has blue cubes, green cubes and brown cubes.
The two sides need to balance.
Put the cubes on the balance.
Draw a picture to show your balance scales.
Write a number sentence to describe what you did.

Make It Balance

Reasoning behind the task

An understanding of equality as the notion of balance is critical for student success in both number and algebra. This task helps build that understanding. At the same time it allows students to practice addition and subtraction skills.

Because the task is open-ended and does not specify how many cubes to use, students can work with numbers that are comfortable for them. Because the task asks for three colours on one side and two colours on the other, students will have to be more thoughtful than if they could simply duplicate the same numbers on both sides.

Curriculum coverage

- Pattern and relationships
- Addition
- Algebra and equality

Expectations

All	Most	Some
• Are able to draw or visually display cubes to match the given criteria for small numbers of cubes. • Begin to be able to write number sentences.	• Recognise what the relationship between the colour combinations has to be and correctly create several correct number sentences to make this happen. • Are able to use one solution to create several others.	• Recognise what the relationship between the colour combinations has to be and correctly create many correct number sentences to make this happen. • Are able to use one solution to create many others. • Create situations to match given constraints.

Make It Balance

Key questions

- Was it possible to use one of each colour? Why or why not?
- Could you use the same amount of two different colours? How?
- Could there be more red cubes than blue ones? How?
- Could there be more yellow cubes than green and brown ones together? How?
- Once you had one solution, how could you use that to help you get another one?
- Could there be a lot of cubes on the left and only a few on the right? Why or why not?
- What do you have to do to keep the scales balanced?
- Suppose you used 2 blue, 2 green and 2 brown cubes on one side. What could you do on the other side?
- How could you use adding and subtracting to help you?

Scaffolding learning

- Use or draw a set of scales and / or use real cubes.
- Arrange the cubes on the scales making sure the colours in are the correct places.
- Start off with a small number of cubes.
- Use more cubes - think about the different ways you could arrange them.
- Draw a picture or your balanced scales and write a number sentence for each solution.

Challenge

Change the rules so that the left side of the balance is MUCH lower than the right side, or the right side needs to include more than 6 different colours of cube.