One Two

Series A

Rich Learning Tasks

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Problem Solving and Reasoning

Contents

Name:

Number

Walking Forwards and Backwards

Choose a place on the number line.

Decide how many steps to move each time:

Go forward _____ steps.

Then back ______ steps.

Then forward ______ steps.

Then back ______ steps.

You have to land where you started.

Show your journey on the number line.

Did your number get bigger or smaller?

Have another go.

1	2	3	4	5	6	7	8	9	10

Walking Forwards and Backwards

Number

Reasoning behind the task

This task provides students with an opportunity to practice the counting sequence, observe the effect of moving forward and backward on a number line, and explore, informally, the inverse relationship between addition and subtraction.

Although some students may assume you must go backward as many steps as you go forward each time to get back to the start number, they will come to realise that it is simply the total forward and the total backward that must match.

Curriculum coverage

- Number
- Number lines
- Counting

Expectations

All	Most	Same
• Solve the problem using trial and improvement, using physical objects such as counters or moving themselves.	• Reason out at least two solutions by realising that moving forwards and backwards by the same amount each time works.	• Reason out many solutions realising that moving forward in total the same amount as moving back in total works. • Work without using any physical objects to count with and work beyond the constraints of the number line.

Key questions

- If you went forward 2, back 2, forward 3 and back 3, would you land where you started? Why or why not?
- If you went forward 2, back 3, forward 3 and back 2, would you land where you started? Why or why not?
- If you started at 4, could you go forward 2, back 3, forward 4 and back 4, and land where you started? Why or why not?
- If you had started at a different number, could you have still taken the same number of steps?
- Could you pick any three numbers to start with and then choose the fourth one so that you land where you need to?

Scaffolding learning

- Stand on 5. Where would you be if you went forward 1 step? Why is that easy to predict?
- Imagine you go forward 2 and want to get back to where you were. What could you do?
- Imagine you go forward 2 and then back 1. Will you be back where you started? Will you be ahead? Will you be behind?

Challenge

- Restrict students from using the same number of steps in the four movements.
- Ask students to move forwards beyond 10 or backwards beyond 0.

Name:

Ten Balloons

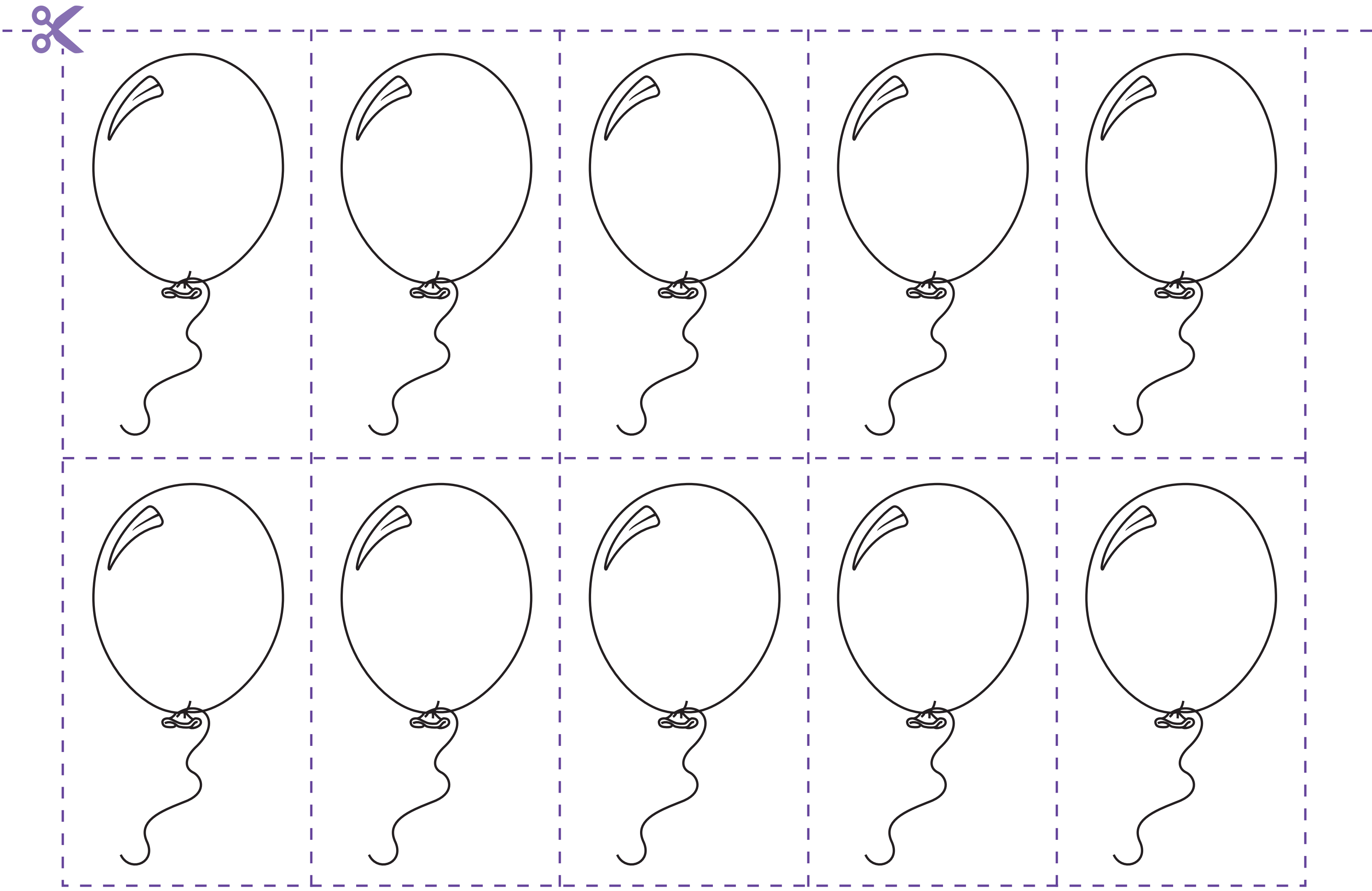
Number

Colour some balloons red, some blue, and some green, using these rules:

- Most of the balloons have to be red.
- The smallest number of balloons have to be green.

State how many of each colour balloon you have created.

Find as many answers as you can.



Ten Balloons

Reasoning behind the task

In this task, students compare small numbers to determine which is greater and which is less. Using the terms most and least in the task familiarises students with the language of comparison. Although the focus is on comparison, students are informally exploring different ways to decompose the number 10.

Curriculum coverage

- Number
- Comparing small numbers
- Partitioning

Expectations

All	Most	Some
• Solve the problem using trial and improvement. • Make some attempt to explain their reasoning.	• Determine several solutions to the problem through reasoning, comparing two numbers less than 10 correctly. • Describe the terms least and most in the context of the problem.	• Reason out the constraints on the number of red and green balloons and determine a number of possibilities recognising those constraints. • Challenge themselves to create more complex rules by adding colours and/or quantity of balloons and solve the problem using reasoning.

Ten Balloons

Key questions

- Could there be 4 red balloons? Why or why not?
- What is the smallest number of red balloons there could be?
- How many green balloons are possible — could it be 5? 4?
- How can you check to see whether your numbers for each balloon are right?

Scaffolding learning

- Think about whether you will choose to colour red, blue or green first
- Think about what most and least mean
- There are a total of 10 balloons – does this help you decide how many of each colour to choose?

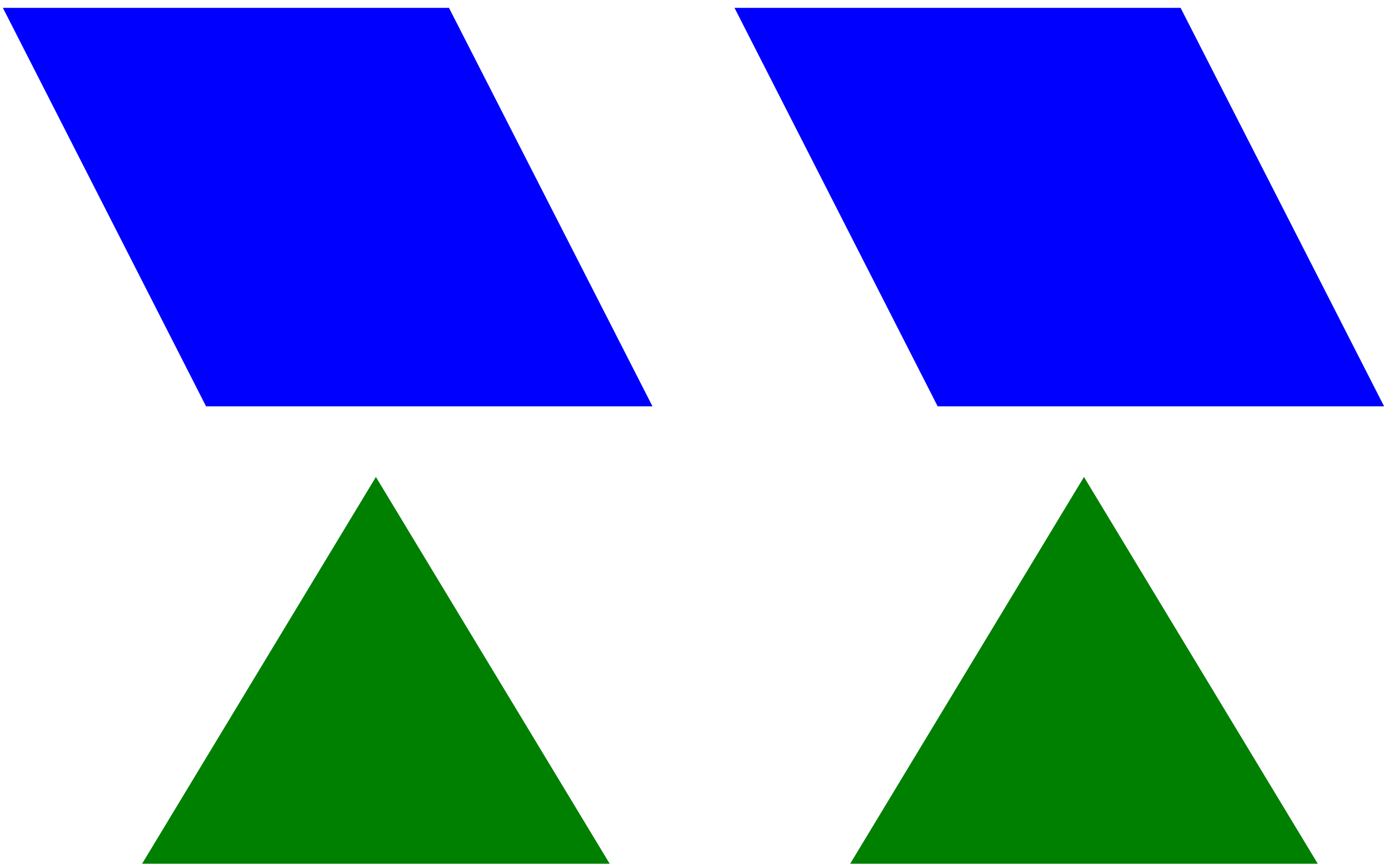
Challenge

- Add 5 more balloons to your picture. How many of each colour will you use now?
- Change the rules and challenge a partner to solve your new criteria, e.g. most of the balloons have to be red, but the number of green and blue ones must be the same.
- Add more colours to include in your new rules.

Name:

Blue and Green Shapes

What shapes or designs can you make using 2 blue and 2 green shapes?



Blue and Green Shapes

Reasoning behind the task

In this task students use shapes to make new shapes and begin to recognise the effect of the orientation of the shapes. They may also begin to think about area and proportionality.

Curriculum coverage

- Geometry
- Properties of shapes
- Exploring shapes

Expectations

All	Most	Some
• Create several different shape designs using the four pieces.	• Create many different shape designs using the four pieces. • Understand that a shape does not change if its orientation changes (a square standing on one of its corners is still a square).	• Create many different shape designs describing the new shape's properties accurately. • Make a hexagon and parallelogram using the shapes.

Blue and Green Shapes

Key questions

- Were you able to put the shapes together to make one big shape or did you create designs made up of separate pieces?
- When all four shapes were put together into one shape, how many sides did that shape have? Was there more than one possible number of sides?
- Choose one of the designs you created. Was it more green or more blue? Why?
- Could you have made the same shapes or designs with only green shapes? How many would you have needed?
- Could you have made the same shapes with only blue shapes? If yes, how many blue shapes would you have needed?
- What could you make if you only used triangles?
- What could you make if you only used one blue and one green shape?
- How would your shape change if you moved one of the triangles to a different spot?

Scaffolding learning

- Cut out the shapes – what are they called?
- Explore rotating them around and arranging all four of them in different ways.
- Place sides and / or corners together.
- Try to make a shape with lots of sides.
- Try to make a shape that does not have any gaps in the middle.

Challenge

- Use 3 or 4 of each shape – what new shapes can you make?
- Make a hexagon or parallelogram. What other shapes can you make?

Name:

Cut it Up

Geometry

Predict which of these combinations would be possible if you cut a rectangle into 3 shapes.

- 3 squares
- 1 triangle and 2 rectangles
- 2 triangles and 1 rectangle
- 3 triangles.

Check your predictions.

Explore whether there are any other ways of doing it.

Cut it Up

Reasoning behind the task

Composing and decomposing shapes is a critical part of geometry. In this particular activity, students use visualisation skills to predict how the original rectangle could be cut. Students also think about names of shapes, different orientations of shapes, and properties of shapes.

Curriculum coverage

- Geometry
- Properties of shapes
- Exploring shapes

Expectations

All	Most	Some
• Make some attempt to predict which are possible and impossible. • Explore through trial and improvement.	• Correctly predict several of the combinations that are possible and impossible, explaining their reasoning. • Demonstrate which combinations are possible by cutting the rectangles.	• Correctly predict all of the combinations that are possible and impossible, explaining their reasoning clearly. • Demonstrate accurately which combinations are possible by cutting the rectangles. • Explain why if creating 2 squares is possible, 3 is not.

Cut it Up

Key questions

- Which combinations did you think would be possible? Why?
- Would a different number of squares have worked? How many?
- Which worked — 1 triangle and 2 rectangles or 2 triangles and 1 rectangle? Why?
- What number of triangles, besides 3, would have been possible? Why do you think that?

Scaffolding learning

- Draw a straight line across one of the rectangles – what shapes would you make?
- Try and make two squares out of a rectangle.
- Explore cutting a rectangle to make triangles – can you make 3? What about 4 or 5 triangles?

Challenge

Explore different ways of cutting rectangles into a range of different shapes.

Name:

Measurement

Baby Steps and Giant Steps

How big do you think a giant step is?

How big do you think a baby step is?

How many baby steps make a giant step?

Show your thinking.

Baby Steps and Giant Steps

Reasoning behind the task

It is important for students to recognise that the value we assign when we measure with a unit depends on two things: the size of some attribute of the object we are measuring and the size of the unit. We might use a lot of units because the object being measured is big, or we might use a lot of units because the unit is small. This task, by not fixing either the unit or the object, allows for both ideas to emerge.

Students might choose to interpret this as an area question, rather than a length question. Those students might end up with larger numbers since they are counting steps in two dimensions and not just one.

Curriculum coverage

- Measurement
- Non-standard units

Expectations

All	Most	Some
• Choose a reasonable baby and giant step and begin to describe how many baby ones are needed for one giant step.	• Choose a reasonable baby and giant step and describes the number of baby steps needed to equal one giant step. • Refer to only one size of unit or object when describing how to make the number of units bigger or smaller.	• Choose a reasonable baby step and giant step size and determine the number of baby steps that are equal to one or more giant steps using a suitable technique. • Refer to the size of the unit and the size of the object when describing how to make the number bigger or smaller. • Solve measurement problems using the units they have developed.

Key questions

- Is it easier to pick your baby step and giant step first, or is it easier to just put together baby steps and decide when you have a giant step?
- How long would two baby steps be? Does that look like a giant step?
- How did you decide when the baby step and giant step were the right size?
- What did you do to decide how many baby steps make a giant step?
- How could you have made your number of baby steps even bigger?
- How could you have made your number of baby steps smaller?

Scaffolding learning

- Think about how big a baby step could be – draw it on the paper.
- How many baby steps do you think would make a giant step?
- Draw the number of baby steps that are the same as one giant step.

Challenge

- Work out how many baby steps there are for 2, 5 and 10 giant steps.
- How many giant steps would be needed to make a King of the Giants' step? How many baby steps would be the same as one of the King of the Giants' step?

Name:

Folding Lines

Draw a red line.

Now draw a blue line that is slightly longer than the red line if you fold it in half.

Reasoning behind the task

This task provides an opportunity for students to compare lengths. It helps to build visualisation skills by asking students to imagine what half or twice as long would be. The task starts students on the road to proportional thinking by relating the concepts of half and twice and by helping them to realise that the problem is not about absolute lengths, but relative lengths. Two students could be correct with very different lines because it is the relationship between the two lines that matters.

Rather than asking that the folded line be exactly the same as the original, the phrase "a slightly longer" is used so that students will not be hung up on being perfect, which is not really what the task is about.
The dotted gridline could be used to support students who are struggling to visualise lines that are a little more than double of the original.

Curriculum coverage

- Measurement
- Comparing lengths
- Proportion

Expectations

All	Most	Some
• Draw a blue line that is a little longer than twice the length of their red line.	• Recognise that their blue line is a little longer than twice the length of the red line and explain why.	• Recognise and describe why their blue line is a little longer than twice the length of the red line and that their red line is a little shorter than half as long as their blue line. • Explain why different people can have different length blue lines linked to the length of their red lines.

Folding Lines

Key questions

- Will the blue line be longer or shorter than the red line? Why / why not?
- Do you think it will be a lot longer?
- Could you just look at your blue line and know it is just a LITTLE longer than the red one if you folded it in half? What would you have to do to check?
- Describe how your red line compares to your blue line in length.
- Would a line that is exactly twice as long as the red line be longer or shorter than your blue line? Explain your thinking?
- Would everyone who does this task end up with the same length line? Why / why not?

Scaffolding learning

- Draw a red line on the page (you could use the dotted gridlines to help you). Make sure your line is less than half the width of the page.
- Think about how long your blue will need to be – will it be the same length, longer, much longer?
- Draw your blue line and check it is a little longer than twice the length of the red line.

Challenge

- Draw a blue line that is a little longer than the red line if it's folded twice.
- Draw a blue line first and then draw a red line that is a little shorter than the blue one when folded once/ twice.

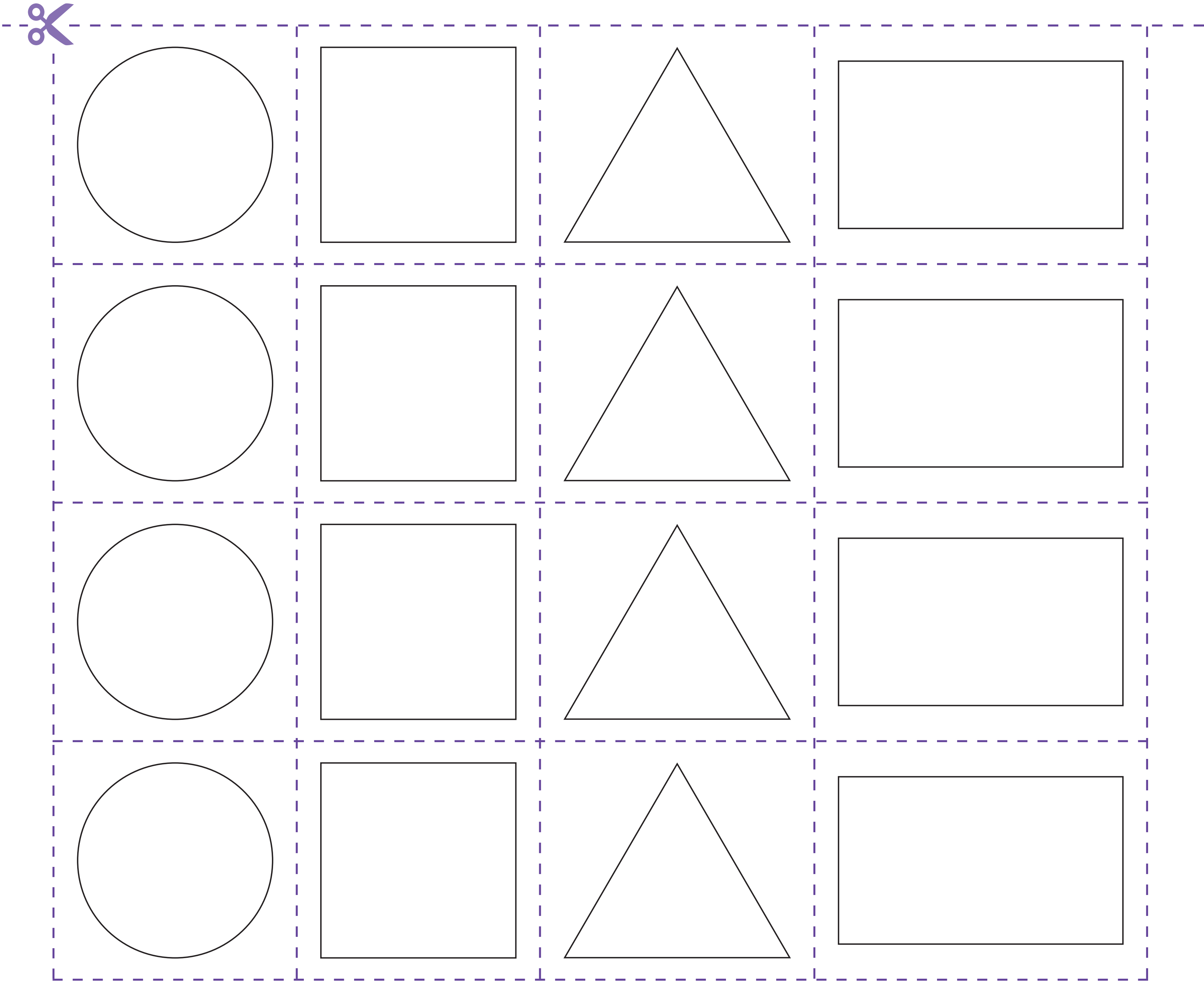
Name:

More Blue

Colour the shapes. Make different patterns that have more blue shapes than green ones.

Which of your patterns do you think are most alike?

Can you make your pattern a repeating pattern?



More Blue

Reasoning behind the task

Young students enjoy creating patterns. Providing many shape options gives students the opportunity to create many different patterns and explore how those patterns are alike and different. They also have the chance to work on shape recognition, but, more importantly, to look at pattern structure. Questioning will focus them on what makes a pattern a pattern—namely its predictability.

Asking students to create patterns that are more blue than green will lead them to use more complex patterns than simple AB ones, and will lead to more reasoning. Students are required to realise that once the core of a repeating pattern has more blue than green there will always be more blue than green, no matter how many times the core is repeated. Some students might realise that even if their core has more blue, if they don't use full repetitions of the core, there could be more green than blue at the point at which they stop showing their pattern, (e.g., if the core were green, green, blue, blue, blue and they showed green, green, blue, blue, blue, green, green). However, this will not be true in the long run.

The students will need several copies of the worksheet to draw their patterns on.

Curriculum coverage

- Patterns

Expectations

All	Most	Some
• Use more blue than green in their pattern and have the beginnings of a repeating pattern.	• Create a repeating pattern with more blue than green in it. • Describe why their patterns are alike focussing on the structure of the pattern.	• Create a repeating pattern with more blue than green, and explain why it is a repeating pattern. • Explain that patterns continue forever and that therefore there will continue to be more blue than green, if the pattern continued.

Key questions

- Does it matter which shape you start with?
- Do you have to start with a blue shape?
- Should you just colour one shape blue and then the next one green, and then repeat that over and over? Why / why not?
- What makes something a pattern? Describe why yours is a pattern
- How have you made yours a repeating pattern?
- How will you make sure your colours and shapes repeat?
- How can you be sure that there will be more blue than green, even if you continue the pattern past where you stopped?
- How did you decide which two of your patterns were most alike?

Scaffolding learning

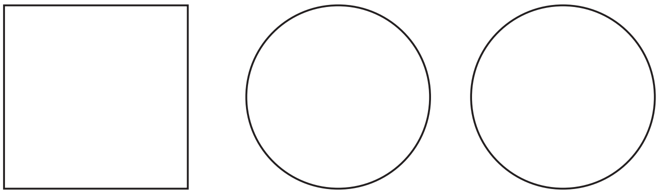
- Choose how many green and blue shapes you are going to colour, making sure you have more blue than green.
- Think about how to arrange your colours – will you block them together or space them out?
- Think about how to make your pattern a repeating pattern.

Challenge

- Make a pattern where for every 3 green shapes, there are 4 blue ones.
- Make a repeating pattern with.

Lots of Patterns

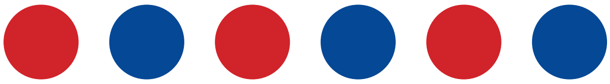
These three shapes are the beginning of a pattern.
Draw lots of different ways the pattern could continue.



Lots of Patterns

Reasoning behind the task

This task helps students realise that knowing the start of a pattern does not determine the pattern; any start can be continued in many different ways to create patterns. No pattern is "set in stone" until someone indicates the rule for the pattern, i.e. what is the core or how the pattern grows. Even the pattern below can be continued many ways.



Although most people assume it continues alternating red and blue circles, it could well be that what is shown is followed by three alternating red/blue pairs of squares, then back to circles, then squares, etc.

Some students will believe that the only way to continue the pattern in the task is repeating the same three shapes they see, but encourage students to think of other ways as well.

Curriculum coverage

- Patterns

Expectations

All	Most	Some
• Continue the pattern in one or more ways using the shapes that are already there (circles and squares).	• Continue the pattern in at least two unique ways, including shapes that are not already present, e.g. including a triangle or rectangle. • Describe how their pattern is a repeating pattern.	• Continue the pattern in at least three unique ways, including shapes and colours that are not already present, e.g. including a triangle or adding colours. • Understand that what you see at the beginning of a pattern does not necessarily determine the whole of the core of the pattern. • Explain what makes a pattern a pattern and describe how their pattern is a repeating pattern.

Lots of Patterns

Key questions

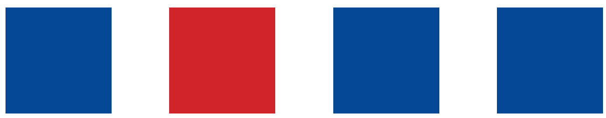
- Could you put a circle next in your pattern?
- Could you put a blue shape next?
- Could you include triangles or rectangles?
- What makes something a pattern?
- What part of your pattern repeats?
- What else could you change besides colour and shape?

Scaffolding learning

- Think about what shape could come next in the pattern. Could it be a square? Could you use a rectangle or triangle?
- Think about adding colour.
- How will you make your pattern repeat?
- Think about how to make a simple or more complex pattern.

Challenge

Create a different pattern start, e.g. the one on the right that is slightly more complex, and continue it many ways:

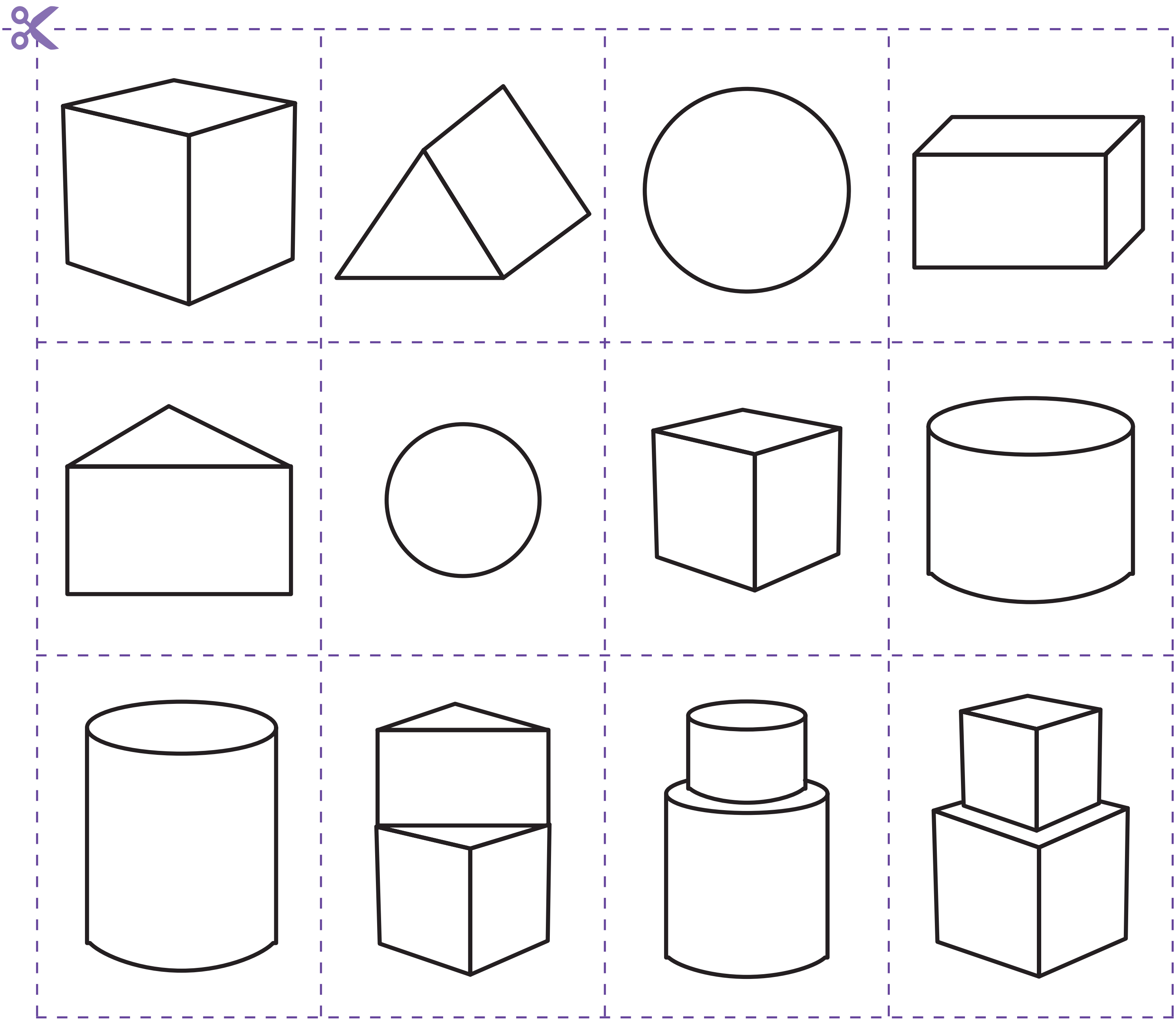


Name:

Who Belongs Together?

Cut out each of the shapes. Choose 4 that you think belong together and put them in a group. Choose a shape that you think does not belong with them. Explain why it doesn't belong.

Repeat this again with 4 different shapes and one that doesn't belong. Use different reasons for why they belong together that you used for the first 4 shapes.



Who Belongs Together

Reasoning behind the task

Sorting shapes is the basis for work in pattern, geometry and measurement. It is by classifying shapes that students can distinguish items to make patterns (e.g. red vs. blue or circle vs. square), can name shapes, and can distinguish between shapes with big and small areas.

This particular task provides students with the opportunity to create their own sorting rules; this gives them more "responsibility" than guessing someone else's sorting rule. By asking students why one shape doesn't belong, we encourage them to be flexible thinkers. By asking them to repeat the task with different shapes, we further encourage flexible thinking.

Curriculum coverage

- Data
- Classifying shapes
- Sorting objects by one or more criterion

Expectations

All	Most	Some
• Sort one or two sets of five shapes using simple sorting rules.	• Appropriately sort the five shapes, justifying why four belong and one does not for at least three different sets of shapes using simple sorting rules. • Begin to use shape properties in explanations.	• Appropriately sort the five shapes, justifying why four belong and one does not for at least three different sets of shapes. • Have a clear understanding of shape properties and use this understanding to describe why shapes belong or not. • Use a wide range of 'sorting rules'.

Who Belongs Together

Key questions

- What are some of the things about shapes that you can use to sort them?
- Choose a shape. What other shapes would go with it? Why do they belong together? What other shape wouldn't belong? Why?
- How did you decide that a shape didn't belong? How many things about the shape did you have to think about?
- Imagine there was a small yellow cylinder and a large yellow cube. What would make them go together? What would stop them belonging together? What shapes wouldn't belong with them?
- Is it possible for a sphere to belong with a cube? How?
- Is it possible for a triangular prism to belong with a cylinder? How?

Scaffolding learning

- Choose a shape. Think about its properties: How many sides does it have? Is it 2D or 3D? How many edges or faces does it have?
- Find 3 shapes that you think belong with your shape and one that does not. Explain your thinking behind your decisions.

Challenge

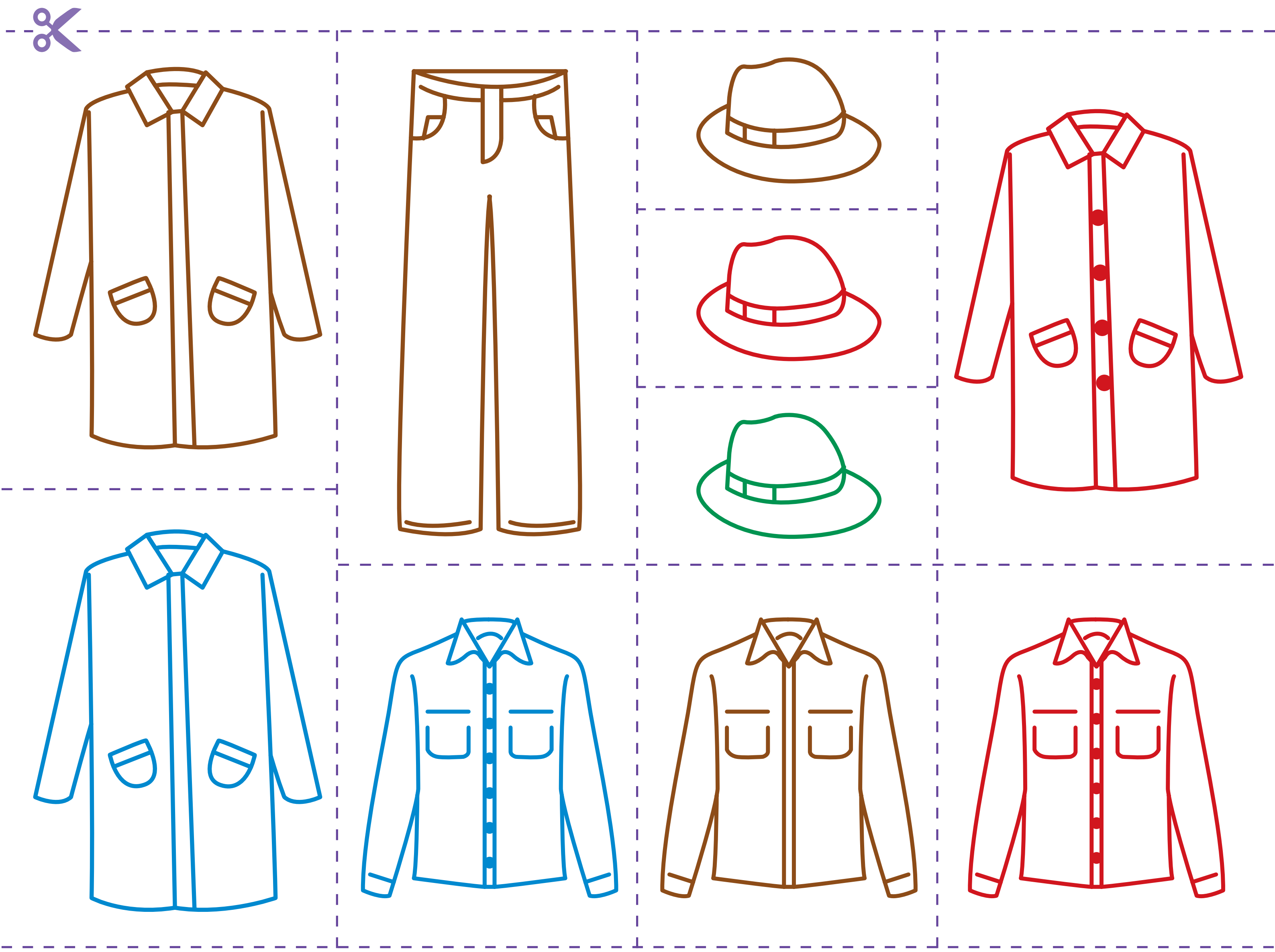
Use a wider range of shapes. Ask students to describe their reasons for shapes belonging or not using detailed understanding of shape properties.

Name:

Split Them Up

How can you sort the clothes into three groups that go together so that one group includes 4 things, and the other two include 3 things each?

Is there more than one way to do this?



Split Them Up

Reasoning behind the task

This task has a problem-solving element in requiring students to create certain size groupings as a result of the sort. A student cannot just randomly create categories, but has to think more carefully about what the possibilities could be. He or she also has to think more broadly so that what might be perceived as separate categories (such as trousers and hats or yellow things and green things) could go together.

Curriculum coverage

- Data
- Sorting objects by one or more criterion

Expectations

All	Most	Some
• Sort items using the correct group sizes using simple categories.	• Sort the items creating the correct group size groupings, describing each category without simply naming the items in it. • Combine disparate items in meaningful ways.	• Sort the items creating the correct group size groupings in at least three different ways, describing each category without simply naming the items in it. At least one sorting rule does not involve colour. • Combine disparate items in meaningful ways, clearly defining why they could be grouped together. • Explain how to add items without ruining the sort.

Key questions

- Could one of the groups be yellow things? Why / why not?
- Were you able to make one group trousers? Why or why not?
- What could the trousers go with to make a group?
- Were you able to make one group brown things? Could your other two groups be based on colour? Why / why not?
- Could you make a group that was not about colour and not about whether it was clothes?

Scaffolding learning

- Cut out the objects and lay them out.
- Think of different ways you could sort them.
- Arrange them into three groups, one of 4 and two groups of 3.
- Think about what category you are using to sort the items.
- Can you arrange them in different ways that match the criteria?

Challenge

Remove the group size rule. How could the items now be grouped? What categories have you chosen? Create your own group size rules and items. Challenge a partner to sort them according to your criteria.