



Series C



Rich Learning Tasks

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Problem Solving and Reasoning

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Name:

Number

More and More

Choose a selection of counters, blocks or teddies and count them.
Now make a second selection choosing twice as many as you had in the first set (you should now have 2 sets of objects, with twice as many in the second set as there are in the first set).

How many do you have now in total?

Repeat this for lots of different selections.

What are the total numbers you could get?

What are the total numbers you could not get? Why not?

Reasoning behind the task

This task asks students to double a range of numbers and then add together the totals to look for patterns. Students should realise that any multiple of 3 is a possible total answer and be able to work out how many would be in the two sets for larger multiples of 3 such as 60.

Asking students both what a number can be and what a number cannot be is an important part of helping them learn to conjecture.

The task allows for differentiation since some students are more comfortable than others with greater numbers, even though using actual objects should help students who struggle.

Curriculum coverage

- Number
- Odd and even numbers
- Doubling

Expectations

All	Most	Some
<ul style="list-style-type: none"> • Find at least two possible total answers that are multiples of 3 by adding sets together. 	<ul style="list-style-type: none"> • Find several examples of total numbers that work. • Notice that total answers are multiples of 3 and begin to explain why these work. 	<ul style="list-style-type: none"> • Understand that all multiples of 3 are possible as total numbers, and all other numbers are not possible. • Predict correctly larger total numbers that are possible giving a clear reason why.

Key questions

- Is it possible to get both small numbers of total objects and large numbers of total objects?
- Once you have one possible answer, could you use this to help you get other answers? How?
- What is the least number of objects you could have in total? How do you know?
- Could the total number of objects be odd or even or both? How do you know?
- Do you think you could end up with 4 objects? Work backward to try to find out.
- Do you predict it would be possible to get 60 objects or not? Why?

Scaffolding learning

- Start with just one object in set 1, what total number of objects would you end up with? What if you started with 2 or 3 objects in set 1?
- Think carefully about which total number of objects you could and couldn't have.
- What large number of objects do you predict you could have as totals?

Challenge

Explore what total numbers are possible if set 2 has three times or four times more than set 1 each time.

Name:

Coins

Number

Eight coins are worth a little bit less than 7 coins.

What could the coins and their values be?

Write down lots of different possibilities.

Reasoning behind the task

This task provides an opportunity for students to solve money problems and count coins as well as add and subtract. It focuses students on the important concept that fewer coins can be worth more, because different coins have different values.

The task encourages students to make conjectures, try lots of possibilities, and use reasoning as they work out why the coin sets have to be different and how they can be different if the values are not that far apart. In making those conjectures, students get a lot of practice in adding, subtracting, and counting coins, even though they are solving only one problem. Because the amounts must be close, more refined reasoning is required. Solutions can include a variety of coins such as £1 coins, 50p coins, 20p coins, 10p coins, 5p coins, 2p coins, 1p coins.

Curriculum coverage

- Number
- Money
- Addition
- Subtraction

Expectations

All	Most	Some
<ul style="list-style-type: none"> • Identify at least one way that 8 coins could be worth less than 7 coins, but the values are not particularly close. 	<ul style="list-style-type: none"> • Identify at least three ways that 8 coins could be worth less than 7 coins where the difference in values is close. • Explain why it is possible that fewer coins can be worth more than less coins. 	<ul style="list-style-type: none"> • Identify many ways that 8 coins could be worth slightly less than 7 coins where the values are extremely close e.g. with a difference of only 1p or 2p. • Explain why it is possible that fewer coins can be worth more than less coins, giving several examples going beyond the solutions to the problem.

Key questions

- How can fewer coins be worth more?
- Could you use 8 of the same coin for one group and 7 of the same coin for the other?
- Do you think there will be more valuable coins in the group of 7 coins or the group of 8 coins? Explain your thinking.
- Could you make 3 coins be worth a little less than 2 coins?

Scaffolding learning

- Start off with 1p, 2p and 5p coins.
- Explore arranging them in groups of 7 and 8 coins and add them up.
- How close in value can you get their totals?
- Make 8 coins worth a little less than 7 coins.

Challenge

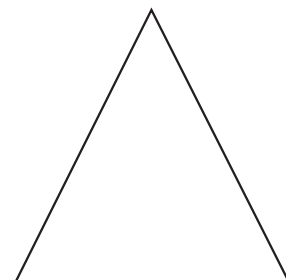
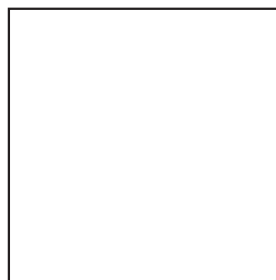
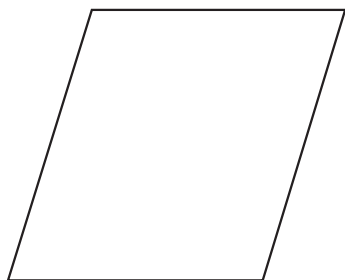
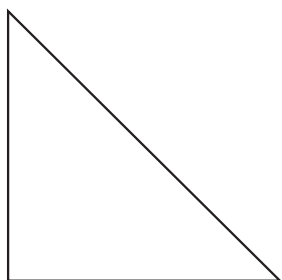
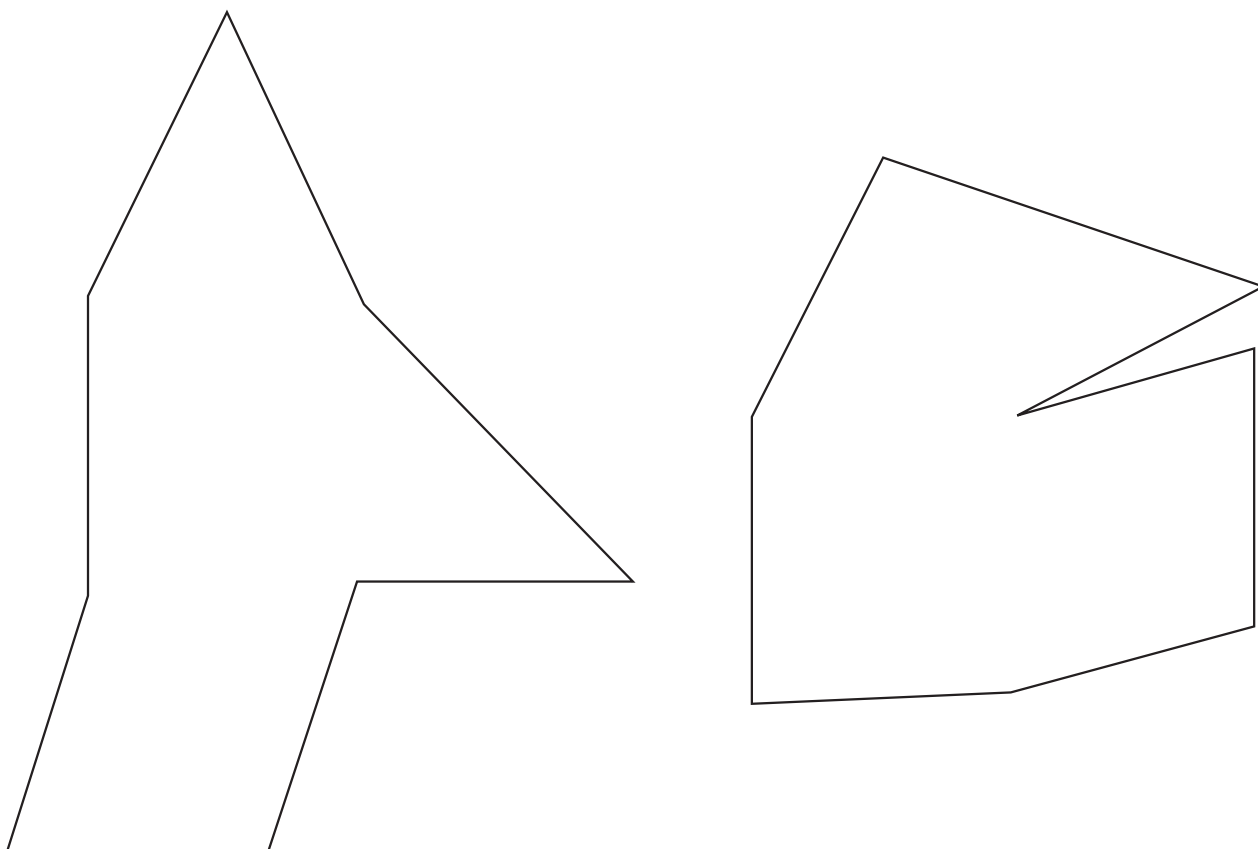
- Make 5 coins worth a little bit less than 8 coins.
- Make 1 or 2 coins a little bit less than 12 coins.

Name: _____

Shape Puzzle

Cut out the shapes at the bottom of this sheet and use them to cover each of the shapes at the top.

What about the big shapes helped you the most to work out where to put the pieces?



Reasoning behind the task

Students gain an opportunity to see how shapes can fit together to create other shapes - a very important geometric skill. They also observe that even if a shape is turned or reflected, it does not change its size or its proportions.

Simple shapes have been used as puzzle pieces to help students see that even very simple shapes can be combined to create quite complex shapes. Some of the composite shapes make it easy to see where to place the shapes and some are not as obvious, to allow for better differentiation.

Curriculum coverage

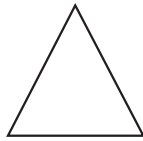
- Geometry
- Shape and space
- Rotation and reflection

Expectations

All	Most	Some
<ul style="list-style-type: none">• Correctly place two or more of the puzzle pieces over one of the big shapes, but don't quite complete the puzzle.	<ul style="list-style-type: none">• Correctly cover at least one of the puzzles exactly using the four puzzle pieces, using trial and improvement.	<ul style="list-style-type: none">• Correctly cover both puzzles exactly using the four puzzle pieces.• Clearly explain what clues they used to help them place the pieces.

Key questions

- How did looking at the corners of the big shapes help you choose what went where?
- The four separate shapes have 16 sides. Why don't the combined shapes have 16 sides?
- There were square corners in some of the original shapes. Why didn't there have to be square corners in the combined shape?
- Was it easier to see where the triangles, the parallelogram, or the square fit? Why do you think that?
- Do you see the parallelogram inside the big shape? Where?
- Do you see this triangle point to the triangle shown below inside the big shape? Where?



- Maybe you could turn the shape or flip it over. Does that help you see it in the bigger shape?

Scaffolding learning

- Cut out the shapes and explore arranging them in different positions.
- Look carefully at the big shapes – try to visualise where the small shapes could fit in.
- Explore turning and / or rotating the shapes to fit them inside the big shapes.

Challenge

Arrange the shapes in your own way to create a new big shape. Draw around the pieces and challenge a partner to work out how the four shapes fit into it. Make it harder by flipping one or more of the shapes over.

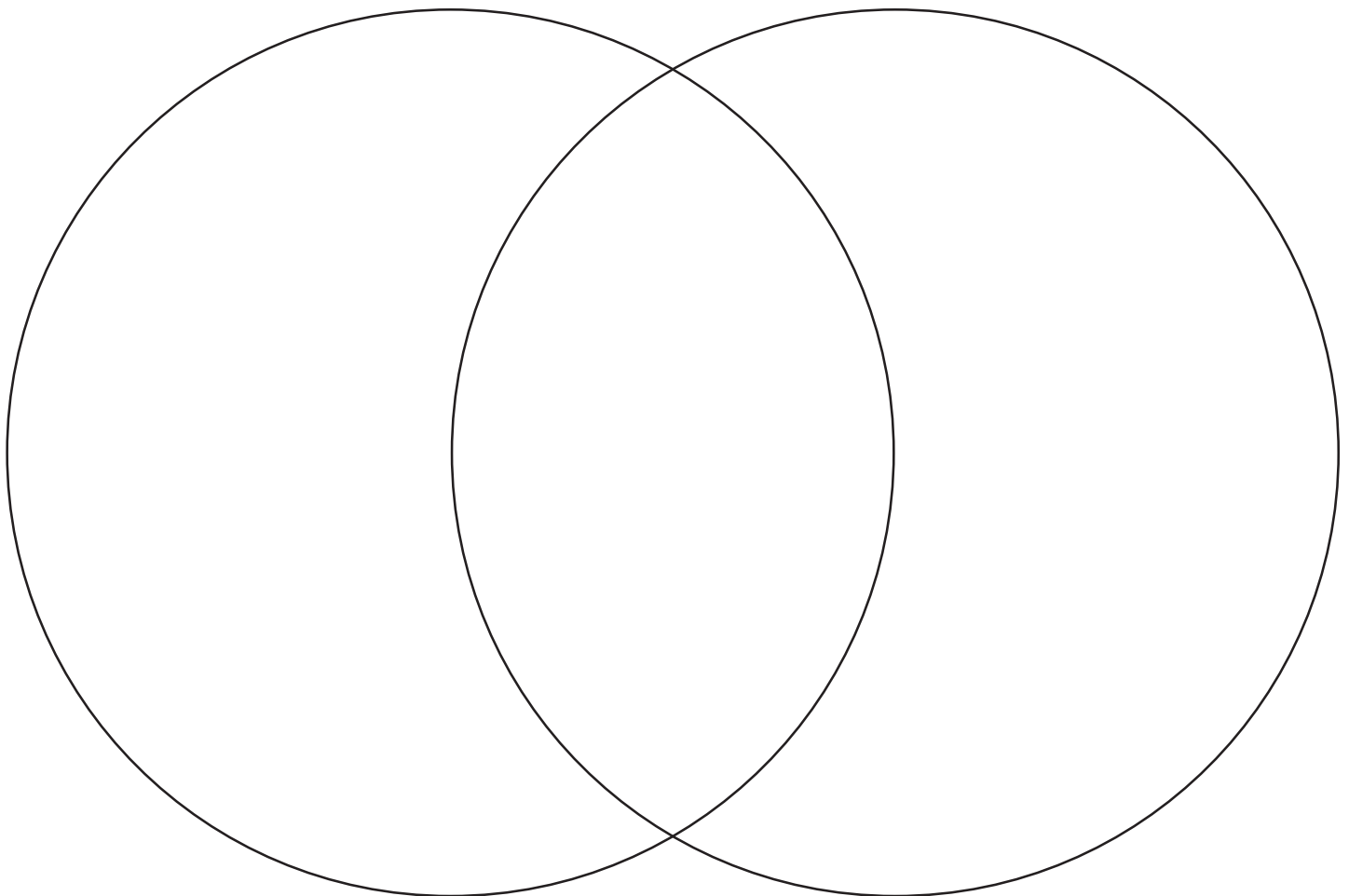
What Goes Where?

You can only put shapes inside each circle that go together for some reason. You can find the shapes on the next page.

Describe what the shapes you have chosen have in common by writing a word for each circle. The word explains what the shapes have in common.

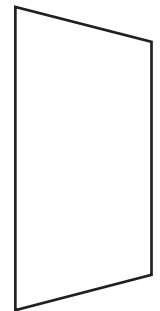
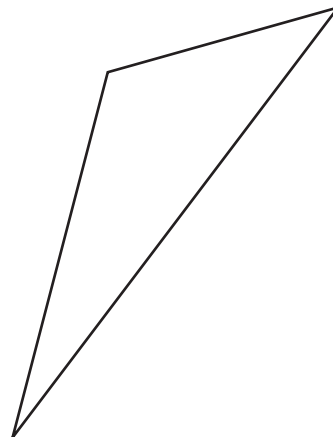
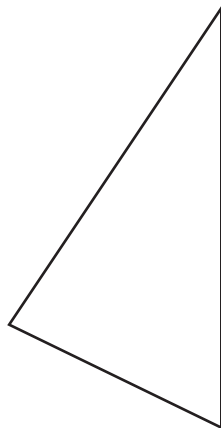
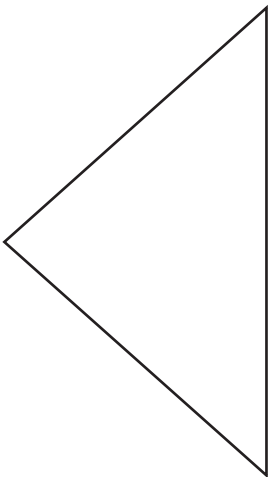
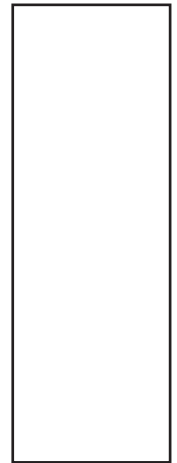
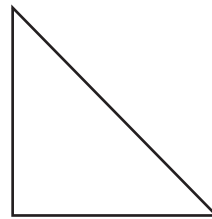
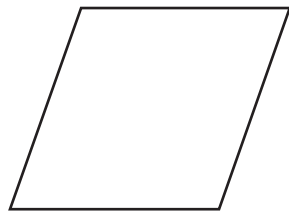
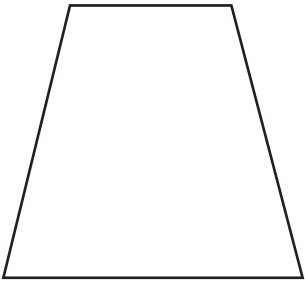
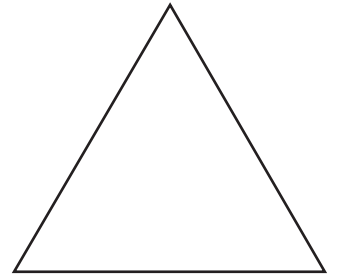
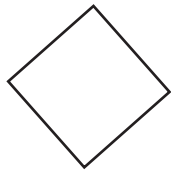
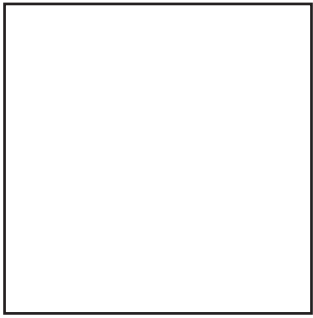
You have to choose words for the circles so that there are shapes in all three parts of the circle diagram. The shapes in the middle have to fit the rule for both circles.

Try to do this in lots of different ways.



Name: _____

What Goes Where?



Reasoning behind the task

Students have a chance to use two attributes of their own choosing to sort a set of shapes. Many options for attributes are available using the provided shapes, including having at least one equal angle, having four sides, and having at least two equal sides.

Students will need to realise that certain choices will not lead to shapes that would be listed in the intersection of the circles; for example, they can't choose one circle for shapes with no equal sides and one for shapes with at least some equal sides.

The use of the Venn diagram (the sorting circles) is not the only way to show a sort using two attributes, but it is a useful graphic organiser for students to use.

Curriculum coverage

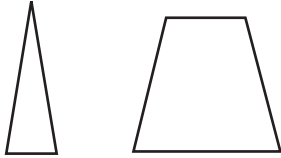
- Geometry
- Sorting objects by one or more criterion
- Properties of shapes
- Venn diagrams

Expectations

All	Most	Some
<ul style="list-style-type: none"> • Sort the majority of shapes according to one or more attribute, in at least one way. 	<ul style="list-style-type: none"> • Correctly label the Venn Diagram and sort the shapes so it includes appropriate shapes in all three sections, at least two different ways. 	<ul style="list-style-type: none"> • Correctly label the Venn Diagram and sort the shapes so it includes appropriate shapes in all three sections and outside the circles, at least four different ways. • Understand that naming a shape is a way to sort shapes. • Identify at least one way that all shapes are alike and different relating to a geometric attribute.

Key questions

- What do you notice about the angles of the shapes that might help you decide which shapes belong together?
- What could be true about a square and a triangle that might make them belong together?
- Why might these two shapes belong together?



- Which shapes have names you know? Does knowing the name of a shape help you work out a way to label the circles in the graph? How?

Scaffolding learning

- Choose any shape you like. Find another shape that is like it in some way. What makes them alike?
- How would you label the circle if you put them both in it?
- Think about shapes you could put in the other circle – these need to be not like the shapes you've already chosen.
- What label could you write for these shapes?

Challenge

Choose two of these geometric attributes to sort shapes together and then create shapes that fit all three parts of the two sorting circles.

Attribute choices:

There are exactly two equal angles.

There are exactly two equal sides.

There are no square corners.

There are more than five sides.

Name:

How Many Units?

You will need:

Paper clips of different sizes.

You are going to measure the red line below with paper clips.

First, choose a paper clip so that it takes A LOT of them to measure the line.

Then choose another size paper clip so that it doesn't take TOO MANY.

Can you choose a paper clip so that it takes more than 3, but fewer than 5 to measure the line?



Reasoning behind the task

The purpose of this task is to reinforce understanding that it takes fewer large units or more smaller units to measure a given length. At the same time, students have an opportunity to use visualisation skills to estimate how many units long a particular length is. Informally, students are using early fractional thinking when they estimate the unit size when just a few units fit.

The line being measured is neither horizontal nor vertical so that students become accustomed to measuring lengths even when they are not straight across or straight up and down. Paper clips from which to choose are not pre-determined; this puts more of the thinking in the hands of the student.

The phrases "a lot" and "not so many" were used to give latitude to students, so that they could come up with different possibilities.

Curriculum coverage

- Measuring with non-standard units
- Estimation
- Proportion

Expectations

All	Most	Some
<ul style="list-style-type: none">• Show that it takes more of the small units than large ones to measure a length, but only relates it to this situation.	<ul style="list-style-type: none">• Demonstrate an understanding that it takes more smaller units and/or fewer larger units to measure a length.	<ul style="list-style-type: none">• Demonstrate an understanding that it takes more smaller units and/or fewer larger units to measure a length, and that there were other solutions than theirs to meet the task requirements.

Key questions

- If you used a big unit (paperclip), would a lot of them be needed to measure the line? Explain your thinking.
- Can you think of something that it would take two of to measure the line? What unit will you use?
- How would you change that unit so that it would take more than 2 but fewer than 3 to measure the line?

Scaffolding learning

- Start off with small paperclips and measure the line with them.
- Next, measure the line with larger paperclips.
- Explore the size of paperclip you will need so that it takes more than 3, but fewer than 5 to measure the line.

Challenge

Attempt to create units to meet other conditions, e.g.:

It takes exactly 4 to measure the line.

It takes more than 1, but fewer than 2, to measure the line.

It takes exactly 2 more units than one of their previous choices to measure the line.

Name:

Hundred Chart

Colour some of the squares in the hundred chart below to make a pattern.

Look carefully at the numbers you have coloured - describe what you notice about the numbers you have coloured.

You might want to add or subtract the numbers if you want to talk about things you noticed about the sums or differences.

Repeat the task with another pattern.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Reasoning behind the task

The organisation of the numbers from 1 to 100 in a 10 by 10 chart has the potential to reveal many properties of numbers. For example, if students colour all the numbers that end in 0, they see the pattern of a vertical line; that's because the numbers are 10 apart. But if they colour all the numbers that are 9 apart, they see the pattern of diagonal lines; that's because 9 more is 1 less than 10 more. If students colour all the even numbers, they see stripes.

Students could start with a visual pattern (instead of starting with a number pattern) and see how the numbers also form a pattern. For example, if they colour the pattern on the right, they will see that within each "V," the top numbers are 18 and 22 less than the bottom number and the middle numbers are 9 and 11 less than the bottom number, and if you add the numbers on the left, you end up with 6 less than if you add the numbers on the right.

The rule for the pattern should be up to the student. It could be every so many numbers, it could be something like the one above that includes sets of a "design," like the V.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Curriculum coverage

- Pattern and relationships
- Skip counting

Expectations

All	Most	Some
<ul style="list-style-type: none"> • Begin to describe the pattern they have chosen for their design. • Identify a relationship between the coloured numbers in their design. 	<ul style="list-style-type: none"> • Create at least two different patterns and explain what the coloured numbers have in common, describing several different observations. 	<ul style="list-style-type: none"> • Describe why their pattern is a pattern making reference to a repetitive element. • Create at least three different patterns and explain what the coloured numbers have in common, describing a range of different observations.

Key questions

- How far apart are numbers that are right above or below each other?
- Where are all the numbers that have a 2 in them?
- What makes your pattern a pattern?
- What if you coloured half the numbers. What might the chart look like? How are the numbers you coloured alike?
- Suppose you wanted only up-and-down lines in your pattern. What would your pattern look like? What would those numbers have in common?

Scaffolding learning

- Try colouring all the numbers in one vertical line. What do you notice about them?
- Explore colouring every 'x' number. What do you notice?
- Colour a design, such as a V shape. Is there a relationship between the numbers at the top of each V?

Challenge

Use a 100 chart that is arranged in rows of 5 or 20 instead of rows of 10 and see how the same patterns play out on this new chart.

Name:

More and More Dots

Here is a pattern of dots that is growing.

There are more dots in each picture.



Picture 1



Picture 2



Picture 3

Follow the pattern to draw picture 4.

How many dots will there be in picture 4?

Make up your own pattern of dots so that there are 10 dots in your pattern's fourth picture.

Make sure you can describe what sequence the pattern follows.

Repeat for several more patterns with the fourth picture showing 10 dots.

Reasoning behind the task

The study of increasing/growing patterns relates well to skip counting, to multiplication, and eventually to the study of linear relationships. Young students can play with patterns to set the stage for more formal study of patterns later on.

Dots were chosen for this task to unveil how visual representations of mathematical situations can provide another way in to looking at number relationships. For example, the pattern of dots shown in the example models 1, 3, 5, ... dots. If the student looked at just the numbers, he or she might see only that they are odd. But the visual reinforces that odd means double something and one more (the horizontal line without the left corner is repeated in the vertical line without the top corner and the corner is added in). It might also make the pattern of adding 2 easier for the student to see—one dot added on the right and one on the bottom.

Because 10 dots are required in the fourth picture, the student cannot just copy the provided pattern, but might still use it as a starting point: for example, adding 3 dots to each of the given pictures would make the 4th picture have 10 dots. Many possible other patterns of dots are possible that would allow 10 dots in the 4th picture, e.g., 4, 6, 8, 10, ...; or 16, 14, 12, 10, ...; or 5, 10, 5, 10,

Curriculum coverage

- Pattern and relationships
- Skip counting
- Addition

Expectations

All	Most	Some
<ul style="list-style-type: none"> • Create at least one repeating or growing pattern where the fourth picture has 10 dots. 	<ul style="list-style-type: none"> • Create at least two different patterns where the fourth picture has 10 dots, at least one of which shows a growing or decreasing pattern. • Describe what makes their pattern a pattern referring to repetitive elements. 	<ul style="list-style-type: none"> • Create at least four different patterns where the fourth picture has 10 dots, at least two of which show growing and decreasing patterns. • Describe what makes their pattern a pattern referring to repetitive elements, explaining clearly how their pattern could be extended and that other patterns are possible.

Key questions

- Explain how the dots in the example form a pattern.
- Would it still be a pattern if you added a dot to each picture? Where would you put the added dots?
- How else could you have made a pattern starting with 1 dot?
- What makes your pattern a pattern?
- How does the number of dots change in your pattern?
- How many dots would be needed in the 5th picture?
- Do you think there are other possible patterns with 10 dots in the 4th picture? Draw them.

Scaffolding learning

- Look carefully at the pattern. Think about how it is growing each time. How have dots been added to each picture? Draw picture 4. How many dots will you need to use?
- Create your own pattern. Start by drawing 10 dots for picture 4. How many dots will you draw for pictures , 3, 2 and 1
- Make sure you can describe what makes your pattern a pattern.

Challenge

- Create 3 more different patterns that have 10 dots in the fourth picture.
- Create a pattern which has 20 dots in the fifth picture or 28 dots in the third picture.

Name:

Brothers

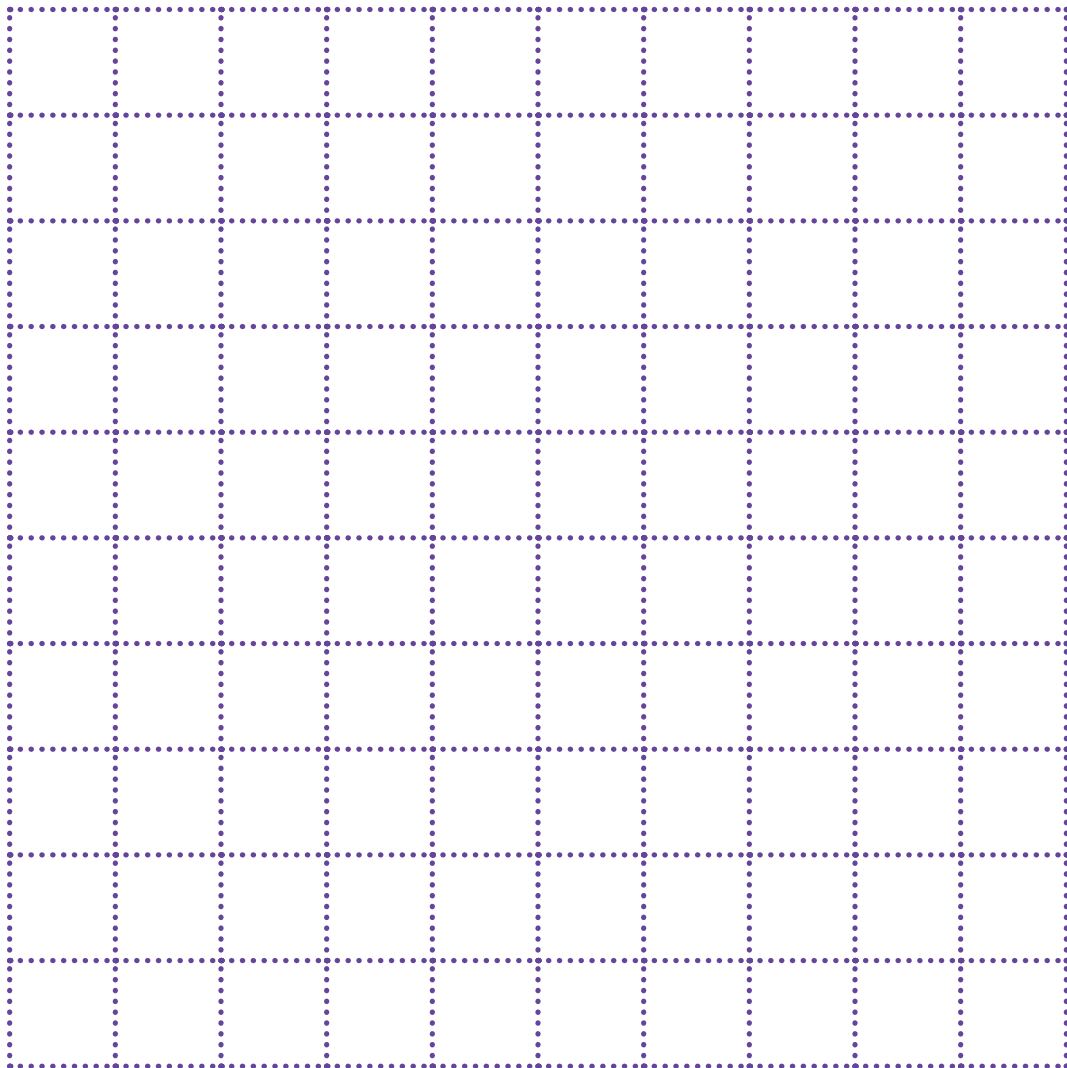
You are going to ask all the other students in your class how many brothers they have and draw a graph to show the results.

What do you predict the graph will look like? How will you present the results?

Make a pictogram of your prediction using the grid below.

What icons will you use in your pictogram?

Now survey the class and record your results. Draw a pictogram of the actual results and compare the two graphs.



Reasoning behind the task

When presented with data, whether in table form or visually, students should always question it and consider whether or not it makes sense. This requires them to think about the type of information being displayed and what they already know about the situation being described.

Most children will realise, through life experience, that it is possible to have lots and lots of brothers, but not many people would have more than, for example, 3 or 4 brothers, and most people would have either 0, 1, or 2 brothers.

Students are asked to create a graph rather than a table to reinforce the value of visual displays. Using pictures as elements of the graph makes the graph more meaningful to young students than simply using rectangles as we do with bar graphs. Asking students to create the graph rather than choosing from some possible graphs puts more of the onus on the students to do the thinking.

Students should be encouraged not to record individual classmate names along the x axis of the graph, but to record the number of brothers, e.g. 0, 1, 2, 3 etc. The y axis of the graph should show how many classmates had that number of brothers. Many students mistakenly believe that the pictures in the graph represent the brothers, but actually the pictures represent the individual children who are surveyed, not the brothers. That means the total number of pictures should represent the total number of students in the class (possibly with the interviewer student missing). One indication of misunderstanding of what the icons represent might be (although not necessarily) that the student uses only boy icons in the graph.

Curriculum coverage

- Statistics and data
- Probability

Expectations

All	Most	Some
<ul style="list-style-type: none"> • Create a predicted and actual data pictogram, making reasonable predictions but perhaps with no attention to the total number of icons used. 	<ul style="list-style-type: none"> • Create a predicted and actual data pictogram, making reasonable predictions and including the correct total number of icons. • Explain the choices they made in creating the graph. 	<ul style="list-style-type: none"> • Create correctly labelled predicted and actual data pictograms, making predictions that are very similar to the actual data, including the correct total number of icons • Explain the choices they made in creating the graph, clearly articulating that each picture represents one of the people surveyed.

Key questions

- What does each picture you put in your pictogram represent?
- What makes a graph a good way to show information?
- Why might it make sense to have a category called "more brothers" after 3 brothers?
- What would putting a picture above 0 brothers represent?
- Imagine more people had 0 brothers than 2 brothers. How would that show in the graph?
- How many pictures will we need to use in our graph? Why?

Scaffolding learning

- Think about your classmates and what you already know about how many brothers they have.
- Think about what the possible answers they could give for the question 'How many brothers do you have?' Where on the graph will you put these options?
- Make your predictions for how many of your classmates have each number of brothers and record this on your pictogram using picture icons.
- Label the graph axis and give the graph a title.
- Survey the class and create a pictogram of the actual data. Compare your predictions and the actual results.

Challenge

Create a pictogram showing how many books each class member has read in the last month.

Name:

What is this About?

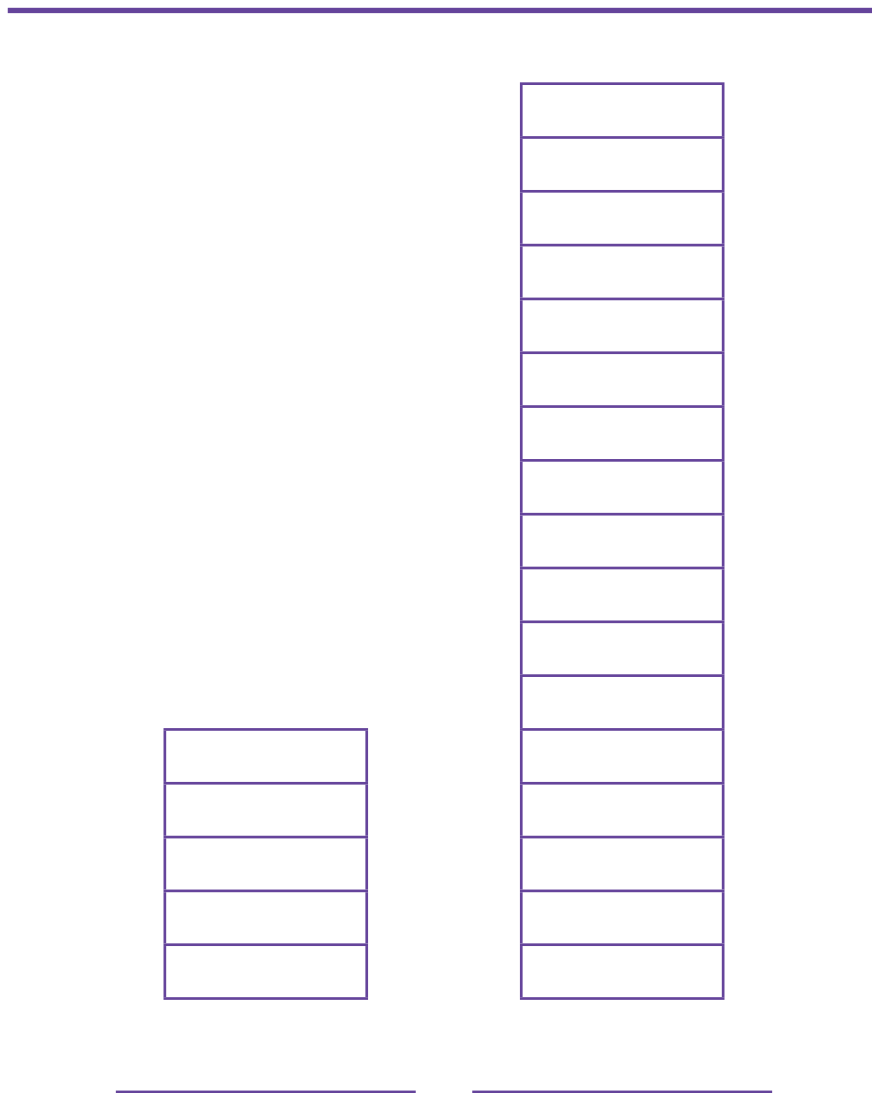
What might this graph be about?

Explain what you think by writing a title for the graph and a title for each bar.

Make sure your ideas make sense.

Write down 4 things that the graph tells you. You could compare the size of the bars and refer to the number of sections in each bar.

Can you think of any other things the graph could be about?



Reasoning behind the task

Usually we provide a topic and ask students to build a graph to provide information about that topic. In this more open-ended activity, the student looks at pre-collected data and realises it could realistically apply to many different situations. Becoming aware of this helps students realise why titles and category names are so important—without them, there is no way to know what the graph is about.

At the earliest levels, students tend to only “read” graphs, generally listing the frequency of each category. But it is important to get them to draw conclusions and make inferences as well. Because students are required to write four things about the graph rather than only two, they must go beyond just reading the information. One of the things they might draw conclusions about is the number of participants in the survey—this is tricky if the categories allow for overlap, but if they do not allow for overlap, it is clear.

The bars were made deliberately quite different in height so that students would draw on their life experience to realise some categories would make more sense than others. For example, if the topic were about who had brothers vs. who had sisters, you might anticipate a more even split, even though it is never certain. A total of 22 entries were used so students might find it reasonable to relate the graph to surveying a class of children at school.

Curriculum coverage

- Statistics and data

Expectations

All	Most	Some
<ul style="list-style-type: none"> • Read the graph and describe what they see. • Discuss a range of data that could apply to some aspects of the graph. 	<ul style="list-style-type: none"> • Read and make inferences about the graph. • Identify a suitable set of data that the graph could represent, labelling the title and bars appropriately. 	<ul style="list-style-type: none"> • Identify a number of suitable sets of data that the graph could represent, labelling the title and bars appropriately. • Understand the difference between continuous and discrete data and know that bar charts represent discrete data.

Key questions

- Imagine the graph is about pets. What could the labels at the bottom be? Who do you think might have been asked? What could the title be?
- Imagine the graph is about food. What could the labels at the bottom be? Who do you think might have been asked? What could the title be?
- Could the graph be about data collected from your class? Why / why not?
- How many people were surveyed in total? How do you know?
- Can you think of anything else that this graph might be about?
- Why are titles and labels so important on graphs?

Scaffolding learning

- Have a look carefully at the graph. What do you notice about the bars? How many entries are there?
- Think about what the graph could be about. What could it not be about?
- Try labelling the bars and giving the graph a title. Do the results in the graph make sense with these labels?
- Write down four things that the graph tells you.
- What else could the graph be about?

Challenge

Imagine a graph with three categories, two very close in size and one with a much higher frequency. Draw this graph and write an appropriate title and labels.