One Two

Series D

Rich Learning Tasks

Dr. Marian Small

Problem Solving and Reasoning

Contents

Name:

15 Blocks

Number

You can represent a number using exactly 15 base ten blocks.
What could the number be?

15 Blocks

Reasoning behind the task

It is important that students recognise that numbers can be represented in many ways. Because this task requires 15 base ten blocks, students have the chance to work with numbers with no more than 9 ones or tens or hundreds or thousands, as well as numbers represented with more than 9 of some unit. For example, the number 24 could be represented with 1 ten rod and 14 one blocks.

Curriculum coverage

- Number
- Partitioning
- Addition

Expectations

All	Most	Some
• Create at least three numbers that can correctly be represented by 15 base ten blocks and correctly identify those numbers.	• Create more than three numbers that can correctly be represented by 15 base ten blocks and correctly identify those numbers. • Identify the lowest number possible and be able to find some fairly high numbers.	• Recognise which of those numbers could also have been represented by a different number of blocks and predict what kinds of numbers those would be and how many other blocks might have been used.

15 Blocks

Key questions

- What is the least number you were able to represent with 15 blocks?
- What is a fairly high number you were able to represent with 15 blocks?
- What is the greatest number of tens you could use? Why?
- How many numbers could be in the 800s? Why only those?
- Could any of the numbers you created have also been represented with a different number of blocks? How many blocks could you have used instead?
- What did you notice about the digits of the numbers you created?

Scaffolding learning

- Have base ten resources available.
- Explore what numbers you can create with fifteen of each type of base ten block, e.g. 15 flats.
- Explore further by using combinations of blocks. Use a methodical approach to finding different possibilities.
- Look for relationships between the numbers you created and the number of blocks you used.

Challenge

Find numbers that can be represented using 24 base ten blocks. Can any of these numbers also be represented using 15 base ten blocks?

Answer: Many (not all) of the numbers using 24 blocks can also be represented using 15 blocks. E.g.132 can be represented by 12 ones and 12 tens (24 blocks) or 2 ones and 13 tens (15 blocks).

Name:

Number

Getting Change

You have bought a gift for your sister. You paid £10 and received 3 coins and a note in change.

How much could the gift have cost?

How many possibilities can you find?

Getting Change

Reasoning behind the task

Being able to solve problems involving money is clearly a useful task. What makes this problem interesting is that students have enough choice in deciding what notes and coins they get back that the problem is appropriate for almost any student. For example, students can choose to have all the coins the same to make the problem easier, or they can consider other possibilities to make the problem more complex.

Curriculum coverage

- Number
- Money calculations

Expectations

All	Most	Some
• Identify one or more possible amounts of change.	• Identify both the possible change and the possible cost of the gift.	• Identify both the possible change and the possible cost of the gift, and recognise why certain costs are impossible.

Getting Change

Key questions

- Could you get a £20 note back in change? A £10 note?
- What choices are there for the note you get back? For the coins?
- What choices of coins would make the problem really easy for you? Why?
- What is the least amount of money you could get back?
- What is the greatest amount of money you could get back?
- Could you get back an exact amount of pounds? Why or why not?
- Could your price have been £[ ]. 50? Why or why not?

Scaffolding learning

- List the possible notes and coins you could have received in change.
- Choose three coins and one note and find their total. To make it easier, choose three of the same coins. Then consider what the cost of the gift would have been should this have been the change given from £10.
- Explore different combinations of coins and different possibilities of gift price.

Challenge

If you paid £20 for a gift and received 2 notes and 6 coins in change, are there many more possibilities for the cost of the gift than in the original problem?

Name:

We Belong Together

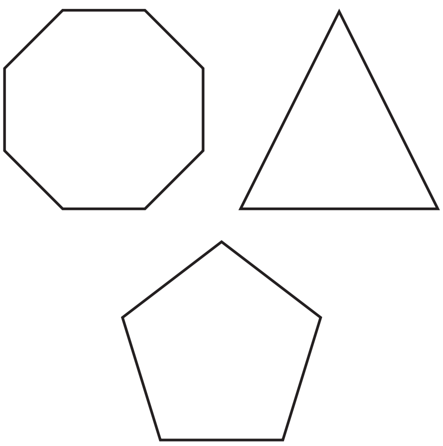
Below are three groups of shapes. Decide what properties the shapes have in common within each group.

Draw two more shapes that could also belong in the group and explain why.

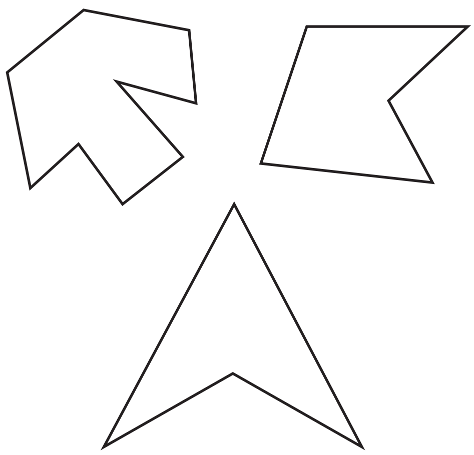
If you can think of more than one reason why the shapes belong together, draw two more shapes that belong with them.

Now draw your own group of five shapes that don't all look the same, but belong together and explain why.

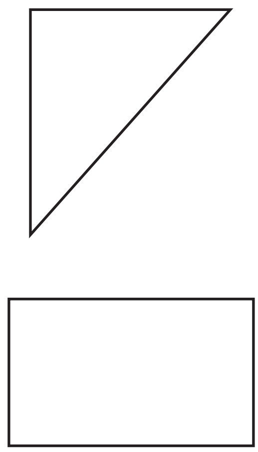
Group 1



Group 2



Group 3



Reasoning behind the task

Shapes have many attributes students can use in describing or comparing them. Rather than asking for shapes with particular attributes or asking whether a given shape has a given attribute, it is more interesting to allow students to hypothesise on how shapes were sorted.

Often there is more than one possibility. For example, the first set of shapes could be considered shapes with all equal sides or shapes with all equal angles, or perhaps shapes with lines of symmetry. The second set of shapes could be considered shapes that are concave (dented in) or shapes with more than 3 sides. The third set of shapes could be considered shapes with at least 1 right angle (although the right angle was deliberately turned not to be in the bottom corner in one case) or shapes with fewer than 8 sides.

Allowing for or even encouraging alternate possibilities is helpful to students. Having students create their own shapes promotes flexibility of thinking.

Curriculum coverage

- Geometry
- 2D shape properties (including angles)
- Sorting objects by one or more criterion

Expectations

All	Most	Some
• Suggest a generic shape property. • Add an additional shape(s) to the groups.	• Suggest a reason why the shapes in each group belong together and adds two appropriate shapes each time. • Create a new set of shapes that belong together. • Show awareness of the different shape properties to consider when sorting.	• Create a new set of shapes that belong together using specific criterion. • Know different shape properties to consider when sorting them.

We Belong Together

Key questions

- Are these shapes unusual? In what way?
- Would looking at how long the sides are be useful?
- Would looking at the angles be useful?
- What properties of shapes do you think are most important to pay attention to?
- Why else might the shapes in Group 1 go together? Group 2? Group 3?

Scaffolding learning

- Consider properties of shapes (number of vertices, length of edges, concave, regular/ irregular shape, angle type, lines of symmetry etc).
- Look for properties that all the shapes share within each group. E.g. Group 1 is a group of regular shapes.
- Add two more shapes to the group that share this common shape property.
- Can you think of more than one shared property within each group?

Challenge

Select and change one of the shapes slightly in each group and suggest a new reason why the two other shapes and the new one belong together.

Name:

Two Triangles

One triangle is a LOT taller than another, but they have the same perimeter.

What is that perimeter and what do the triangles look like?

Did the size of the perimeter affect what you created? Explain.

Two Triangles

Measurement

Reasoning behind the task

It is important for students to learn that shapes that look quite different can still have the same perimeter. It helps students to think of taking a length and "bending" it into a shape. Because one triangle is required to be a lot taller than the other, the student is likely to use a number that is not too small as the perimeter.

Although neither triangle need be isosceles or equilateral, students tend to favour those kinds of triangles. Questioning below will focus them on other types of triangles as well.

Curriculum coverage

- Measurement
- Perimeter
- Triangles

Expectations

All	Most	Some
• Draw two triangles with the same perimeter where the height of one is much greater than the others and correctly calculate the perimeter.	• Draw two triangles with the same perimeter where the height of one is much greater than the others, correctly calculate the perimeter, and describe the shape. • Understand that there are further alternatives to the two triangles drawn.	• Draw two triangles with the same perimeter where the height of one is much greater than the others, correctly calculate the perimeter, and explain the shape dimension choice. • Understand why there are further alternatives to the two triangles drawn. • Describe the transformation of one of the triangles into one with a wide base by recognising that it would have to have a small height. • Start off with a perimeter length and bend it into different shaped triangles.

Two Triangles

Key questions

- If you chose a perimeter of 20 units, could the tall triangle have a length of 10 units?
- What might its dimensions be?
- How could you make a shorter triangle with the same perimeter? What would you have to change about the two triangles you created?
- Did the tall triangle have to have any equal side lengths? Why or why not?
- Could the widths of the triangles have been similar? Why or why not?
- How could you create a triangle that is very wide, with the same perimeter as the two you used?

Scaffolding learning

- Know what the perimeter of a shape is.
- Know the properties of different types of triangles.
- Choose a total length for the perimeter and divide the total into three possible lengths for each triangle.
- Draw your triangles, keeping two sides as long as possible for your tallest triangle (drawing an isosceles for the taller triangle and an equilateral triangle for the shorter triangle will make it easier).

Challenge

Create shapes other than triangles with the same perimeter as the two triangles.

Name:

Clocks

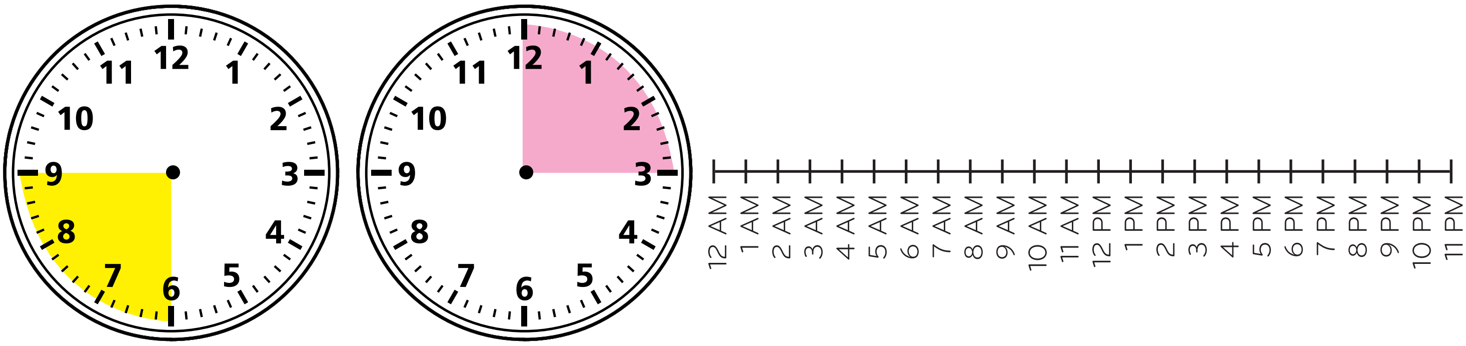
Draw hands to set the first clock to a start time where both hands are in the yellow part of the clock.

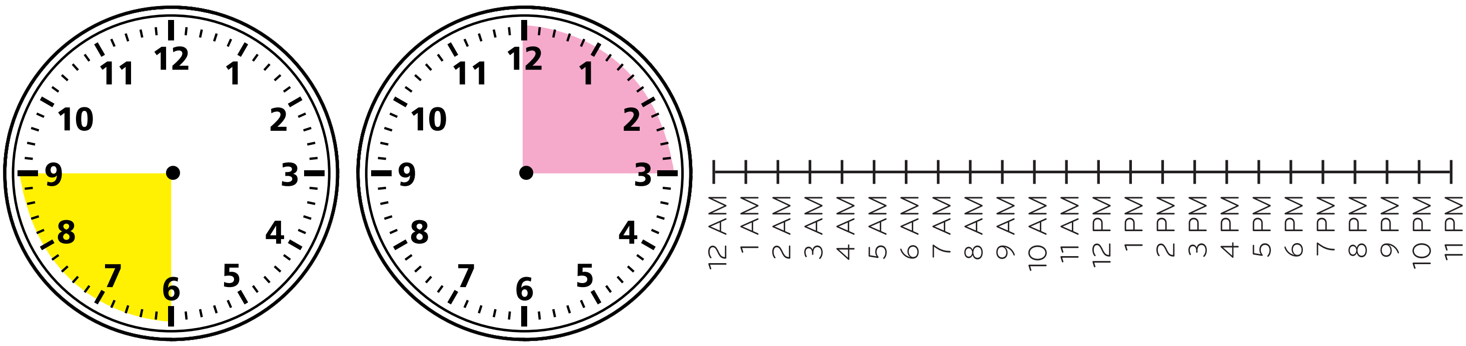
Draw hands to set the second clock to an end time where both hands are in the pink part of the clock.

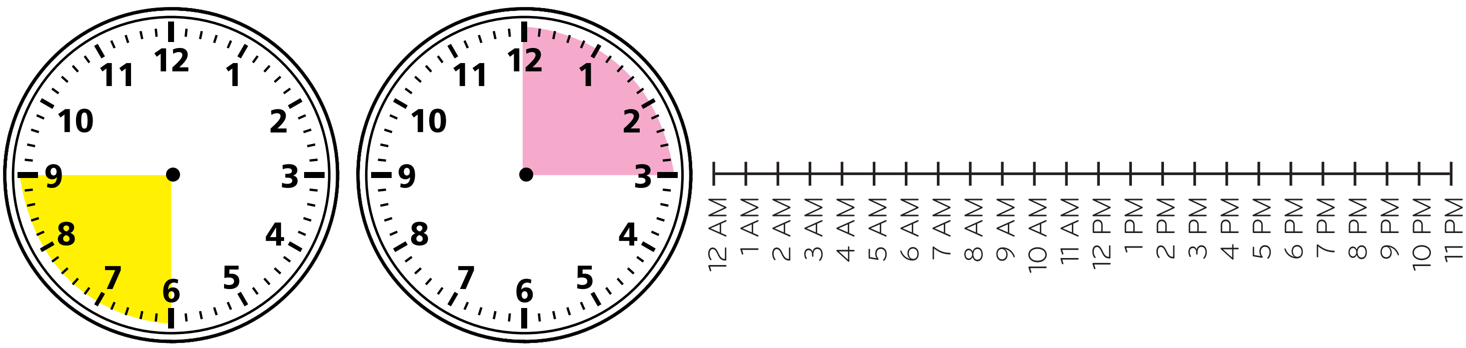
Calculate how much time has passed between the first time and the second time.

Repeat with at least two different times.

START END







Reasoning behind the task

Calculating the amount of time that an event lasts is an important life skill. Many students struggle to determine elapsed time when noon or midnight is crossed, and particularly when the times are not on the hour or half hour. This task is set up so that the initial time has to be something like 6:42 or 7:38, and the final time is on the other side of the hour: e.g., 1:12 or 2:05. Because students are allowed to choose their own times, they can stick with multiples of 5 minutes or even half-hour intervals if that makes them more comfortable.

Students should be encouraged to use number lines or the clocks with moveable hands to help them calculate elapsed time.

Curriculum coverage

- Measurement
- Compare and measure time
- 12-hour clocks

Expectations

All	Most	Some
• Set their clocks with hands on the 12 numbers of the clock, e.g. half hour intervals. • Begin to describe the correct amount of time passed.	• Set their clocks with hands on the 12 numbers of the clock and correctly identify the amount of time that has passed. • Identify the least and greatest amounts of possible time that have passed.	• Set their clock where the minute hand is not on the 12 numbers of the clock and correctly identify the amount of time that has passed. • Identify start and end times so that requirements for particular elapsed times are met.

Key questions

- How much time has passed if you move the hour hand from halfway between 6 and 7 to halfway between 7 and 8?
- What time would it be after 3 hours from your start?
- Why did the amount of time from start to end have to be more than 3 hours?
- What is the longest time it could have been? How do you know?
- How might you have set your clocks to make it $5\frac{1}{2}$ hours from start to end?
- Could the time have been 4 hours and 45 minutes? Explain.

Scaffolding learning

- Look at the clock faces. Draw the clock hands onto the shaded sections of the two clocks. Understand that drawing hands positioned on half hour intervals, or the clock numbers, will make finding the difference between the times easier. Challenge yourself by choosing times which are not on the clock numbers, e.g. 6:42, or 1:12
- Use number lines, or clocks with movable hands, to calculate the difference between the two clock times. Think about how you will move along your timeline/clock to calculate the time difference, e.g. counting in 5, 10, or 30 minute intervals. Think of methods for recording this accurately.

Challenge

Identify all the possible start and end times if the time passed was 6 hours and 22 minutes.

Name:

Balancing Act

All of the yellow boxes hold the same number of cans.

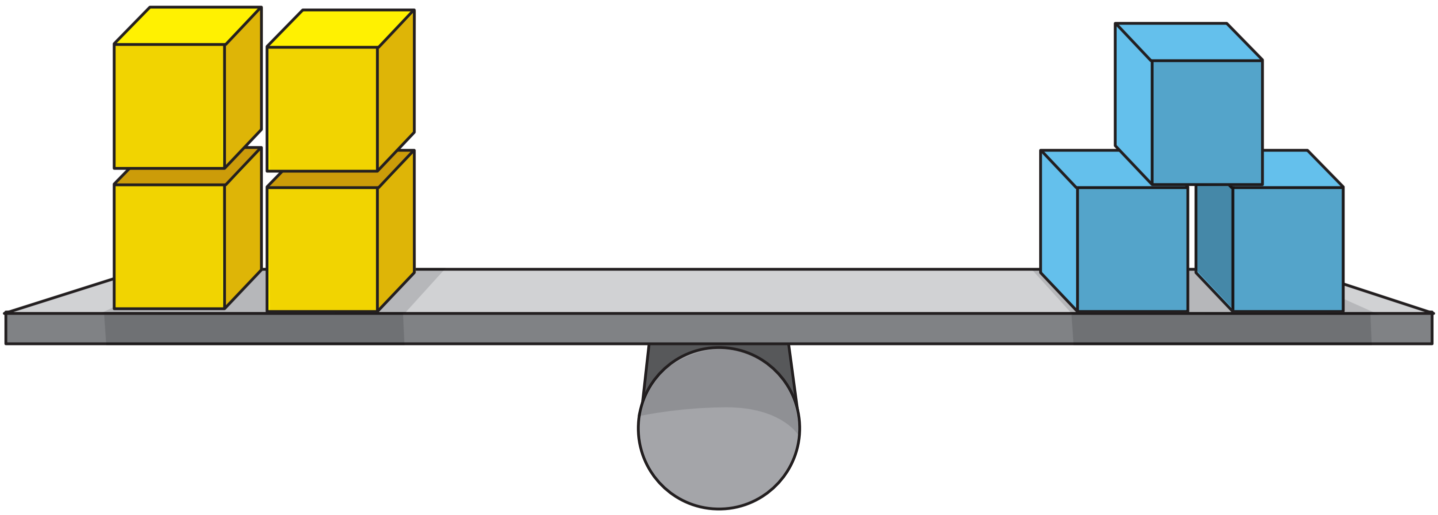
All of the blue boxes hold the same number of cans.

How many cans could be in each colour of box?

How do you know?

Think of lots of possibilities.

How are the number of cans in the two colours of boxes related?



Reasoning behind the task

It is essential for future development in algebra that students have an appropriate understanding of equality—in particular, that equality represents a balance. In this case, because no numbers are given, the students have the opportunity to come to a generalisation about the relationships between the numbers of cans in each type of box. Coming to a generalisation is a big part of what algebraic thinking is all about.

If four yellow units match three blue units, students need to realize that the yellow unit must be $\frac{3}{4}$ of the blue unit. There are many ways to see this:

- If yellow is $\frac{3}{4}$ of blue, each blue is a yellow with an extra fourth. The three extra fourths make another yellow, so there would be 4 yellows matching 3 blues.
- If yellow is $\frac{3}{4}$ of blue, the total of yellows is $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ blues = $\frac{12}{4}$ (or 3) blues.
- If each yellow box held 3 cans, there would be a total of 12 cans. If 12 cans were shared among the 3 blue boxes, each blue would hold 4 cans, and 3 is $\frac{3}{4}$ of 4. A similar thing would happen if the yellow box held 6 cans or 9 cans.

Students are essentially solving a multiplication problem: How can three times one amount be the same as four times another?

Curriculum coverage

- Pattern and relationships
- Fractions and ratio
- Algebra and equality

Expectations

All	Most	Some
• Identify a possible value for how many cans could be in each colour box and begin to justify with reasoning.	• Identify more than one correct value for how many cans could be in each colour box and explain thinking.	• Identify that, in general, the number of cans in the yellow box is $\frac{3}{4}$ of the number in the blue box and can explain why. • Realise that the number of cans in the yellow box must be a multiple of 3 and the number in the blue box a multiple of 4.

Balancing Act

Key questions

- Which colour box holds more cans? How do you know?
- Do you think the blue box holds twice as many cans as the yellow box? Why or why not?
- Could a blue box hold 10 cans? Why or why not?
- What kinds of numbers describe what a yellow box can hold? Why those numbers?
- What fraction of the number of cans in the blue boxes is the number in the yellow boxes?

Scaffolding learning

- Think about the information given and that it means that the scale is balanced (yellow unit must be $\frac{3}{4}$ of the blue unit).
- Express the relationship between yellow and blue boxes as a ratio (4:3). Use your knowledge of multiples to find possibilities for the number of cans in each colour box.
- If each yellow box held 6 cans, would this help you solve how many cans are in the blue boxes? Remember to keep it balanced – whatever you do to one side of the scale you must do to the other.

Challenge

Explore different combinations of yellow and blue boxes on each side of the balance and identify how the number of cans in those two colours of boxes would be related.

Name:

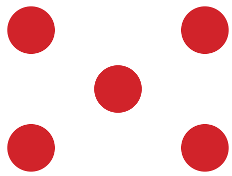
Circle Patterns

Here is the start of an increasing pattern. It has 2 circles and then 5 circles.

Continue growing the pattern until the 10th picture.

Do it lots of ways, making a different pattern each time.

Picture 1



Picture 1

Picture 3

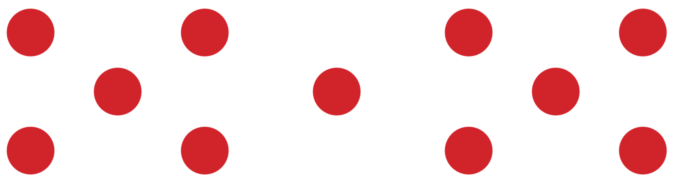
Circle Patterns

Reasoning behind the task

An important notion about patterns is that when no rule is given, there are always many ways to extend the pattern. In this case, students are asked to use different rules to create increasing patterns that begin with the terms 2 and 5. By representing the patterns with shapes, the student often can better "express" the rule. When only two terms are provided, students are likely to find it easier to come up with many ideas than if three terms are given.

Some possibilities include 2, 5, 8, 11, ... (adding 3 each time), or 2, 5, 11, 23, ... (add the number to itself and then add 1), or 2, 5, 9, 14, 20, ... (add 1 more each time), or 2, 5, 8, 12, 16, 21, 26, ... (add the same amount twice, then 1 more twice, then 1 more twice), etc.

Some students might focus on the visual to help them add to the pattern: e.g., noticing that the 2nd term was created by repeating the first term and putting a circle between the original and the copy. This might lead to this as the third term:



Curriculum coverage

- Pattern and relationships
- Addition

Expectations

All	Most	Some
• Create an increasing pattern that starts with 2, 5, showing at least three more terms.	• Create more than one increasing patterns that are different, but start with 2, 5. • Articulate why each pattern is a pattern.	• Create at least three increasing patterns that are different, not starting with 2, 5. • Articulate and identify which pattern grows faster and why.

Circle Patterns

Key questions

- The pattern has to increase. Do you think you want to add, subtract, multiply or divide? Why?
- How are the 1st picture and 2nd picture similar? Different?
- How many rows and columns of dots does the first picture have? The second? How could that be helpful in extending the pattern?
- Which of your patterns grows faster? Do you think it will keep growing faster? Why?
- Could your pattern include the number 10? How?

Scaffolding learning

- Know what a number pattern is.
- Look at the first and second picture in the given pattern. Look for any similarities and differences, perhaps using the number of rows or columns to help you.
- Decide what the next number in the pattern could be and record as a picture. Continue, following the same pattern, recording each number as a picture until the 10^{th} term.
- Describe one of your patterns; what makes it a pattern?

Challenge

Create different ways to extend the pattern that starts 5, 10,...

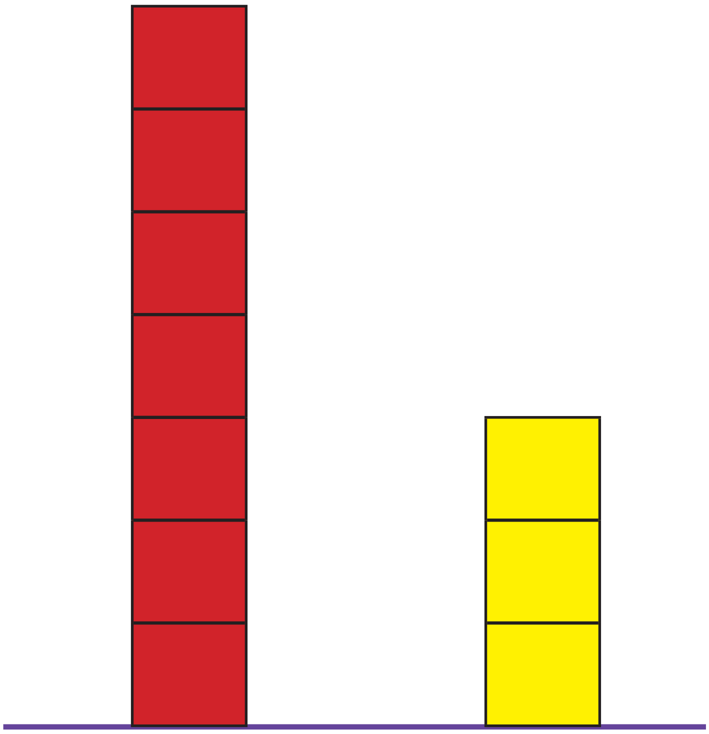
Name:

Sort Us

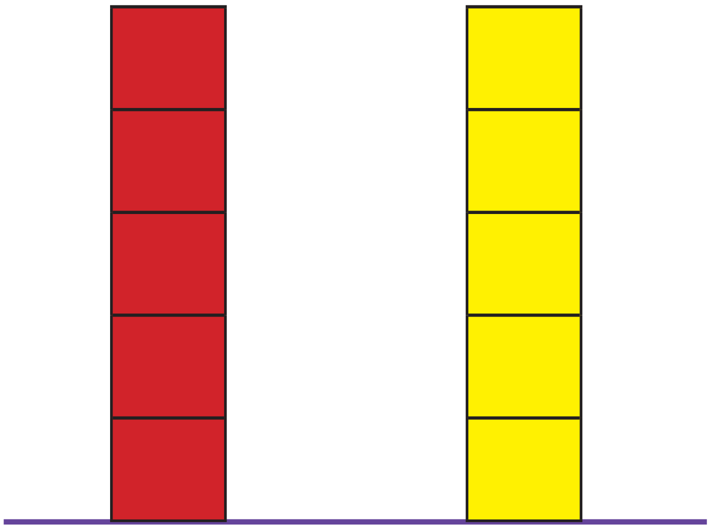
Data

Use the numbers below.

How can you sort them so this graph shows how the 10 numbers were sorted?



How could you sort them to get this graph?



40	121	12	31	9
50	13	220	500	306

Sort Us

Reasoning behind the task

Students frequently gather data and then display it in a graph. In this particular case, the graph is pre-made so that students need to figure out how to make the numbers sortable to fit the pre-made graphs. They have to consider the attributes of numbers they might use, such as size, whether they are even or odd, how many base ten blocks it takes to display them, etc. They also might use some combination of attributes. For example, one category could be even numbers less than 400 and the other, the rest of the numbers.

To make it mathematically richer, the task involves sorting numbers rather than everyday objects.

Curriculum coverage

- Statistics and Data
- Sorting by one or more criterion
- Number facts

Expectations

All	Most	Some
• Create a reasonable way to sort the numbers into one of the given graphs.	• Create reasonable ways to sort the numbers into each of the two graphs, and suggest at least one other way of sorting that would result in a different graph.	• Create reasonable and complex ways to sort the numbers into each of the two graphs, and suggest at least two or more other ways of sorting that would result in different graphs. • Realise that any two numbers could belong together and show how.

Sort Us

Key questions

- Is it possible for 40 and 12 to go together? In what way are they alike?
- Is it possible for 40 and 31 to go together? In what way are they alike?
- Would thinking about the sizes of the numbers help sort them?
- Is there always a way to sort numbers so that any two can go together? Explain.
- Is there anything that was true about only three of the numbers? How would that help you solve the problem?
- Was there a different way that you found to split the numbers into two equal groups?

Scaffolding learning

- Looking at the numbers presented. Think of as many categories for sorting the numbers as possible, e.g. size, whether they are even or odd, how many base ten blocks it takes to display them, whether they are less or more than 30 etc.
- Look at the two graphs and the number of blocks in each bar. Choose a category to sort the numbers by and test to see if it fits the number of blocks. Continue until you identify two categories which can be applied to each graph, e.g. Graph One: Numbers more than 30/Numbers less than 30.

Challenge

Create a random set of 10 numbers and look for all the possible ways to sort the numbers into two bars.