One Two

Series E

Rich Learning Tasks

Dr. Marian Small

Problem Solving and Reasoning

Contents

Name:

Number Line Spacing

There are 4 numbers on a number line. They are labelled A, B, C, and D. All the numbers are greater than 50.

B and C are twice as far apart as A and B.

C and D are four times as far apart as B and C.

a) List 5 possible combinations for the numbers that A, B, C, and D could be. What do you notice about how far apart A and D are?

b) What values do A, B, and C have if D is 100?

Number Line Spacing

Number

Reasoning behind the task

This task provides an opportunity to think of multiplication as comparison—in this case, comparison of length. Students are free to choose how far apart points A and B are, allowing for either simple or more complicated multiplications. Hopefully, some students will come to the generalisation that points A and D have to be 11 times as far apart as A and B.

For example, if a student chose A and B to be 3 apart with A at 60, they will have to do lots of calculation to figure out why D would have to be at 93. But if they had chosen A as 100 and A and B as 5 apart, D would be 155. Asking students to work backwards from D at 100 adds to the computational value of the problem.

Curriculum coverage

- Number
- Multiplication

Expectations

All	Most	Some
• Correctly determine the position of A, B, C and D where the gap between A and B is small. • Notice that there is a gap of 11 between A and D when the gap between A and B is 1.	• Correctly determine the position of A, B, C and D for a range of start A and B points. • Generalise that A and D are 11 times as far apart as A and B.	• Correctly determine the position of A, B, C and D for a range of start A and B points, including fractions, decimals and very large numbers. • Generalise that A and D are 11 times as far apart as A and B and use that information to identify possible and impossible location combinations when D = 100.

Number Line Spacing

Key questions

- Why were B and C an even number of units apart?
- Could B and C have been an odd number of units apart? Why or why not?
- Could A and D have been an odd number of units apart? Explain your thinking.
- What do you notice about the values of D – A?
- How could that have helped you solve the placement of A, B, and C if D were 100?

Scaffolding learning

- Think about the possible start positions for A (it has to be over 50)
- Consider how far apart you want A and B to be. If they are close together it will be easier.
- Place C and D
- Repeat four more times
- Look closely at the values for D – what do you notice?

Challenge

- Choose a large gap between A and B.
- Use fractions, decimals or very large numbers for A.

Name:

Number

Leftovers

You have some counters.

You put them into groups of 3, and there is 1 counter left over.

If you put THE SAME counters into groups of 4, there are 3 counters left over.

a) How many counters could you have?

b) How many different ways can you find to do this?

c) How many counters might you have had if the total number of counters was more than 50?

List as many possibilities as you can.

Leftovers

Reasoning behind the task

In this problem, students deal with division with remainders, even if they don't initially realise that's what they're doing. In fact, they are looking for a number that leaves a remainder of 1 when divided by 3 (a number 1 more than a multiple of 3) that happens to leave a remainder of 3 when divided by 4 (a number also 3 more (or 1 less) than a multiple of 4). It turns out that there are an infinite number of answers: 7, 19, 31, 43, ... continuing with numbers that are 12 apart. The numbers are 12 apart because using an extra 12 counters makes perfect groups of 3 as well as perfect groups of 4, so the remainder is not affected.

Theoretically, students with minimal skills in multiplication could still attack the problem by simply creating their groups and counting the counters they used. But students with multiplication/division skills are likely to use those skills to be more efficient.

When students are asked for a number greater than 50, they are likely to work a little harder than if numbers like 7 and 19 are allowed.

Curriculum coverage

- Number
- Multiplication
- Division with remainders

Expectations

All	Most	Some
• Find one or more possible solutions using counting or visual grouping strategies.	• Find one or more possible solution using counting or visual grouping strategies, extending to mental strategies to find a solution using over 50 counters. • Identify that solutions are all odd numbers.	• Find several solutions including using over 50 counters using mental strategies. • Explain why solutions have to be an odd number and why the gaps between solutions are 12 apart.

Key questions

- What is the least number of counters you could have? How do you know it is least?
- What other ways can you find to do it?
- What did you notice about how far apart the possible answers are?
- Why do you think that happened? Explain your thinking.
- Why could the number of counters not be 60?
- Can the number be an even number? Why/why not?

Scaffolding learning

- Start with a small number of counters and lay them out to help visualise the groupings.
- Look for a number that leaves a remainder of 1 when divided by 3, and a remainder of 3 when divided by 4.
- Think about how to add groups of counters so that the remainder is unaffected (look for a number that can be divided equally by 3 and 4).

Challenge

Change the rules for the problem so that possible answers could be these: 8, 43, 78, 113, ...

Name:

Adding with Blocks

You add a 4-digit number you can represent with 20 Dienes base-ten blocks to a 3-digit number you can represent with 15 blocks.

Represent the answer with 17 blocks.

What could the numbers you added be?

List lots of possibilities.

Adding with Blocks

Reasoning behind the task

Aside from providing an opportunity for students to practice adding, this task requires lots of reasoning. Many students will assume that if you add a 4-digit number using 20 blocks to a 3-digit number using 15 blocks, the total has 35 blocks; and, of course, it could. But the fact that it requires only 17 blocks means that regrouping was used twice (e.g., 10 hundreds became 1 thousand, losing 9 blocks, or 10 tens becomes 1 hundred, losing 9 blocks). The least possible answer is 1133 (which can be shown as 11 hundreds + 3 tens + 3 ones); the greatest has to be less than a 6-digit number since adding a 4-digit number to a 3-digit number cannot yield more than a 5-digit number.

Curriculum coverage

- Number
- Addition
- Partitioning

Expectations

All	Most	Some
• Represent one or more addition sums with 20 blocks for the 4-digit number, 15 blocks for the 3-digit number and 17 blocks for the answer.	• Represent several different addition sums with 20 blocks for the 4-digit number, 15 blocks for the 3-digit number and 17 blocks for the answer, demonstrating a clear understanding of place value.	• Represent many addition sums with 20 blocks for the 4-digit number, 15 blocks for the 3-digit number and 17 blocks for the answer, clearly explaining why it is possible to use 17 or 35 blocks for the answer. • Identify the smallest and largest possible answers, explaining why the answer must be less that a 6-digit number.

Adding with Blocks

Key questions

- What is the smallest / largest number you can make?
- How many different ways can you solve this problem?
- How could you show 100 with 10 blocks? How many different ways can you represent 100?
- Will you have to use thousands blocks in your number? Why/why not?

Scaffolding learning

- Start by laying out blocks to create a 4 digit number.
- Think of a 4 digit number and represent it using 20 blocks.
- Repeat for a 3 digit number and add them together.
- Think about the different ways you can represent the answer using exactly 17 blocks.

Challenge

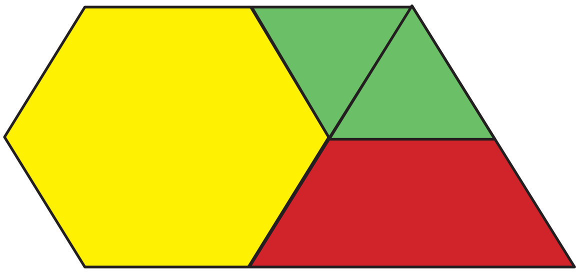
Subtract a 3-digit number represented by 15 blocks from a 4-digit number represented by 20 blocks and end up with a number represented by 23 blocks. How many different addition sums can you write that follow this rule?

Name:

Mirror, Mirror

Where could you put a mirror on this design to see each of the designs or shapes shown below?

What other shapes or designs can you see by putting the mirror at other places on the original design?



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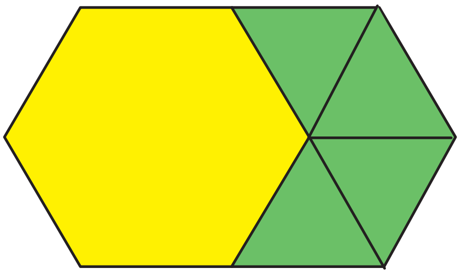

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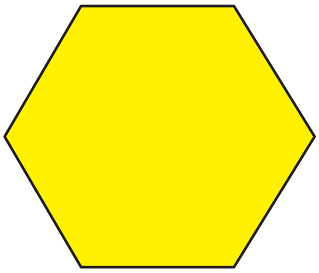

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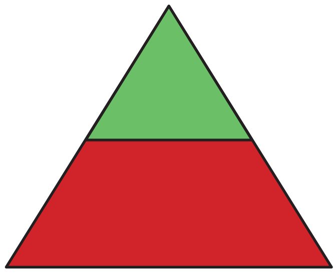

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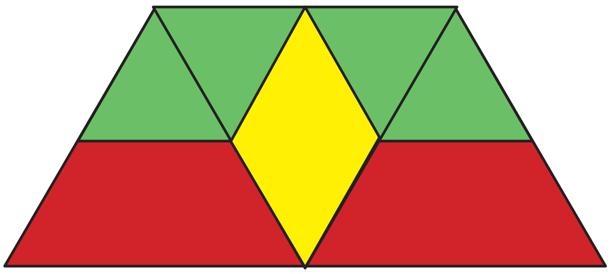

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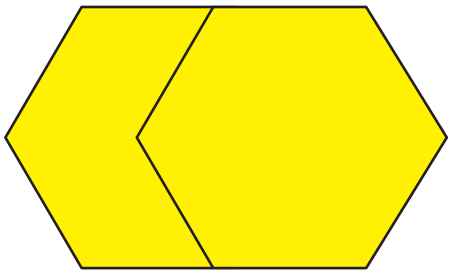


Mirror, Mirror

Reasoning behind the task

Instead of simply asking students to locate lines of symmetry of simple shapes, a more complex design is used and mirrors are applied to it to create new shapes/designs based on reflecting parts of the original shape. Sometimes the mirror line cuts off lots of what is there, and other times it adds to what is there by reflecting to create more.

Curriculum coverage

- Geometry
- Symmetry

Expectations

All	Most	Some
• Explore the design and use the mirror to replicate the designs/shapes shown by placing the mirror accurately, looking into the correct side of the mirror.	• Replicate the designs/shapes shown and explain how to create a design with particular features, e.g. no green pieces/three green pieces.	• Replicate the designs/shapes shown and explain how to create a design with particular features, e.g. no green pieces/three green pieces. • Create several other possible designs based on the original, and design own mirror design challenge with new shapes for others to solve.

Key questions

- How did you know what side of the mirror to look into to create the design you were trying to make?
- When was the mirror line horizontal? What clues told you this?
- When was the mirror line vertical? What clues told you this?
- Why did all of the designs that were given have lines of symmetry?
- What would you see if you put the mirror across the middle of the design? Does the side you look through influence what you see?
- What could you do to eliminate the red part of the original design?
- What could you do to see two red trapezoids instead of only one?

Scaffolding learning

- Explore the design by placing the mirror in different places. What do you see/not see?
- Try to make the shape very small or large, or get rid of some of the colours using the mirror.
- Make each of the shapes by using the mirror to reflect a section of the design.

Challenge

Create your own shape design like the original one, and explore it using the mirror.

Create your own designs using reflections and challenge a partner to work out where the mirror line would go for each one.

Name:

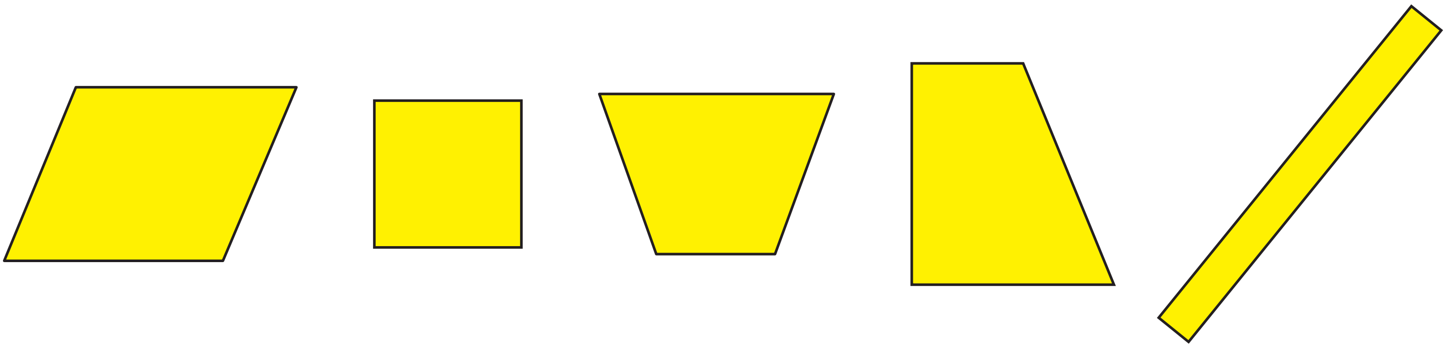
Quadrilaterals

Choose one of the yellow shapes and think carefully about its properties.

Draw a shape that is ALMOST, but not quite, the same as the shape you have chosen. You cannot change the colour. What could you change? Think about length of sides, number of sides, angles, symmetry etc.

Explain why your shape is not quite the same, but almost the same, as the original including information about your new shape's properties.

Repeat for a different yellow shape. Try to use different ideas for each shape you draw.



Reasoning behind the task

This very open-ended task encourages students to consider many different properties of shapes and communicate about them. Because it leaves a great deal of latitude for what makes shapes different, the task should appeal to a wide range of learners, whether strong in spatial reasoning or not.

Shapes might be changed using many geometric properties. Two parallel sides might be made not quite parallel. (Even if parallelism has not been formally studied, students do have an intuitive sense of it with shapes like rectangles and parallelograms.) Two equal side lengths might be made unequal; the symmetry of a shape might be slightly altered. (Even if symmetry has not been formally studied, students have an intuitive sense of it.) A straight side might be slightly bent; a short side might be inserted by cutting off a corner, or a "hole" might be placed in the shape.

Curriculum coverage

- Geometry

Expectations

All	Most	Some
• Create one or more 'new' shapes changing at least one property.	• Create several 'new' shapes considering and explaining at least two different properties they have changed.	• Create many 'new' shapes considering carefully and explaining in detail the many different properties they have changed.

Key questions

- What makes a square a square? What could you change so it would not be a square anymore?
- How could you give your shape more sides without changing it much?
- Could you cut your shape in half?
- Did the side lengths change? Which ones? How much?
- Did the angles change? Which ones? How much?
- Why is your new shape almost the same as the old one?
- What properties of your shape have changed / stayed the same?
- What is the biggest/smallest change that you made?

Scaffolding learning

- Look carefully at the shape you have chosen to change: what properties does the shape have? What minor change could you make to change one of those properties?
- Draw a new shape changing some of the original's properties.
- Explain what you have changed and why it is now different.
- Repeat for other yellow shapes on the sheet.

Challenge

Ask students what names they might give to "almost squares," "almost rectangles," and "almost parallelograms." Ask them to create dictionary definitions for those new shapes.

Name:

Make the Shape

Build a shape that meets the following rules:

There are at least 2 pairs of parallel sides. Make the parallel sides the same colour.
There are 2 or more angles greater than a right angle. Put green dots inside the angles greater than a right angle.
There are at least 2 small angles. Put red dots inside the small angles.

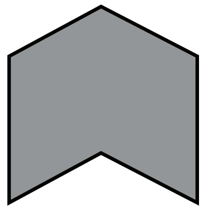
Repeat with at least two more shapes, each with a different number of sides.

Make the Shape

Reasoning behind the task

The focus of this task is on observing and creating shapes with parallel sides. But because students are also required to have small angles in their shapes, they can't just use the standard rectangle, hexagon, octagon, etc. They need to think about how to combine all of the requirements.

A simple solution is a parallelogram but because they have to create a number of shapes, they might end up creating some like the one on the right.



Curriculum coverage

- Geometry
- Properties of shapes
- Angles

Expectations

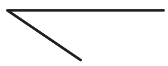
All	Most	Some
• Create one or more shapes that conform to most of the rule criteria, including one or more of the following: at least two parallel sides; at least two correctly marked obtuse angles; at least two correctly marked acute angles.	• Create several shapes that conform to the rule criteria, including at least two parallel sides, at least two correctly marked obtuse angles and at least two correctly marked acute angles. • Understand that it is not possible to make a triangle using the criteria.	• Create several shapes that conform to the rule criteria. • Articulate a strategy for creating further shapes that meet the criteria, eliminating shapes that aren't possible based on their geometric properties. • Understand that any 4+ sided shape is possible and triangles are impossible.

Make the Shape

Key questions

- How do you know your shape can't be a triangle?
- Could it be a 4-sided shape? What kind? Why?
- Does the shape have to have an even number of sides? Why or why not?
- What would be an easy strategy for creating even more shapes?

Scaffolding learning

- Think about parallel lines and how to incorporate at least two sets.
- Try to think creatively and not about familiar or regular shapes.
- Suppose we started the shape like this: 

  What could we do next?

Challenge

- Create a shape that meets the requirements with 6, 8, 12, 15 sides.
- Create your own set of rules for creating a shape and draw three shapes that meet it. Challenge a partner to draw a shape that meets your criteria.

Related Areas

The area of a right triangle (a half-rectangle) is 6 times as much as the area of a rectangle. What could the height and width of the triangle and length and width of the rectangle be?

Create more correct answers if you can.

Reasoning behind the task

Students have a chance to see how changing the length and/or width of a rectangle affects its area.

The area of a right-angled triangle adds an extra dimension to the problem. Since a right-angled triangle is easily viewed as a half-rectangle, there is no need for students to know the formula for the area of a triangle yet.

Since the area of the triangle is 6 times greater, students need to discover that the length x width of the smaller rectangle must be multiplied by 12 to create the larger rectangle of which the triangle is half. This means that it could be done by tripling length and multiplying width by 4, or doubling length and multiplying width by 6, or multiplying the length by 12 and not changing the width, etc.

Because no particular values were specified, students can choose any dimensions they wish for the smaller rectangle and create the associated triangle, or vice versa. There are an infinite number of solutions.

Curriculum coverage

- Measurement
- Area
- Right angled triangles
- Ratio and proportion

Expectations

All	Most	Some
• Create one or more solutions using visual, and trial and improvement strategies.	• Create several solutions using a combination of visual, trial and improvement and mental strategies. • Describe strategies for solving the problem.	• Create several solutions including using large numbers and decimals. • Understand that there are an infinite number of solutions. • Explain that the length x width of the smaller rectangle must be multiplied by 12 to create the larger rectangle of which the triangle is half. • Describe mental strategies for solving the problem.

Related Areas

Key questions

- Does the triangle have to be taller than the rectangle?
- Does it have to be wider?
- Without doing any calculations, what kind of picture could you draw to show that your results make sense? [Students might draw an array of 3 rows of 4 copies of the small rectangle to make a large rectangle, cut the larger rectangle in half, and see why the triangle's area must be 6 times that of the small rectangle.]
- How many small rectangles will you need to equal the area of the right-angled triangle?
- How do you work out the area of a rectangle? How can you use this to work out the area of a right-angled triangle?

Scaffolding learning

- Draw a right-angled triangle and the rectangles you will need to equal its area
- Start off with small numbers for the width and height of the rectangles to make calculating easier
- How many different solutions can you find?

Challenge

If the area of a rectangle was 8 times larger than the area of a right-angled triangle, what could the height and width of the triangle and rectangle be?

Use large numbers or decimals to challenge yourself.

Name:

Measurement

Partial Perimeters

Draw three different rectangles. Choose lengths and widths to make each of these things happen in one of the rectangles.

If you cut the area in half, you lose $\frac{1}{3}$ of the perimeter.

If you cut the area in half, you lose $\frac{1}{4}$ of the perimeter.

If you cut the area in half, you lose $\frac{1}{5}$ of the perimeter.

Try to come up with several possible lengths and widths for each situation.

Partial Perimeters

Reasoning behind the task

Students working on this task are likely to calculate many areas and perimeters of rectangles in pursuit of the required results.

For example, a student who started with a 6 x 9 rectangle with an area of 54 square units and a perimeter of 30 units might cut it in half to form a 3 x 9 rectangle; the new perimeter is 24 units, which is $\frac{4}{5}$ of 30. This would be an example of losing $\frac{1}{5}$ of the perimeter.

Students may end up noticing that the way they cut the rectangle in half could influence the perimeter fraction. For example, if you cut a 6 x 12 rectangle in half to make a 3 x 12 rectangle, the perimeter changes from 36 units to 30 units, a loss of $\frac{1}{6}$ of the perimeter. But if the cut is made to form a 6 x 6 rectangle, the perimeter changes from 36 units to 24 units, a loss of $\frac{1}{3}$ of the perimeter—much more. This previews an important math idea—that longer, narrower shapes have greater perimeters than wider shapes of the same area.

Curriculum coverage

- Measurement
- Perimeter
- Ratio and proportion

Expectations

All	Most	Some
• Find at least one solution for one or more of the three required conditions through trial and improvement.	• Find at least one solution for each of the three required conditions. • Identify that different halvings result in different perimeter losses. • Notice that long, narrow rectangles lose the least perimeter.	• Find several solutions to each of the three required conditions. • Understand that multiplying both dimensions of the rectangle has no effect on the fraction of perimeter lost. • Understand that different halvings result in different perimeter ratios. • Explicitly note that long, narrow rectangles lose the least perimeter and can explain why.

Partial Perimeters

Key questions

- How do you calculate the area of a rectangle?
- How do you calculate the perimeter of a rectangle?
- Why might it be easier to start with a 3 x 4 rectangle than a 3 x 5 rectangle if you have to cut it in half?
- What did you notice about the shapes that lost the least perimeter?
- Does it make a difference if you cut the rectangle horizontally or vertically?
- Suppose a rectangle has a length double the width. What happens to the perimeter when you cut the original rectangle to become a square?
- Suppose you cut it in half the other way, halving the width instead of the length? What effect does this have on the length of the perimeter?

Scaffolding learning

- Start off with a small simple rectangle. Work out its area and perimeter. Explore cutting it in half and working out the new area and perimeter.
- Use trial and improvement to find a solution for losing $\frac{1}{3}$ of the perimeter
- What shape rectangle loses the most/least perimeter when you cut the shape in half?

Challenge

Explore whether it is possible to lose $\frac{2}{3}$ of the perimeter of a rectangle when halving the area.

Pour and Repour

You have a range of small containers which each hold the same fraction of a litre of juice.

When you pour all the juice into one or more big containers, it adds up exactly to a whole number of litres of juice.

How many small containers do you have, and what fraction of a litre might there be in each?

Come up with lots of possibilities.

Pour and Repour

Reasoning behind the task

This task relates to both measuring capacity and adding fractions or multiplying a fraction by a whole number. However, many students who understand the meaning of fractions and who have not met the concepts of adding and multiplying fractions can still be successful with the task using their ability to count up by fractional amounts.

When students are given a choice for the number of containers and the fraction within the container, many more possibilities exist, and a broader range of learners can be successful. For example, a very simple solution is 8 containers holding $\frac{1}{4}$ litre each, for a total of 2 litres. A much more sophisticated answer is 8 containers each holding $\frac{[\,]}{8}$ litres of juice for any value of [] litres.

Curriculum coverage

- Measurement
- Capactiy
- Calculations with fractions

Expectations

All	Most	Some
• Create one or more pictorial representations to solve the problem; begin to write solution as a number sentence.	• Identify several solutions to the problem using simple fractions and write them clearly in a number sentence.	• Identify many solutions to the problem, including a wide range of fractions and whole number amounts. • Clearly articulate the relationship between the fraction of a litre used in each container and the number of containers required.

Pour and Repour

Key questions

- Could each container hold $\frac{1}{2}$ litre? How might this help you solve the problem?
- Could each container hold $\frac{3}{4}$ of litre? If they did, could you end up with 2 containers? Explain.
- Suppose you ended up with 4 litres of juice. How many small containers could you use, and what fraction of juice could have been in each one?
- What do you notice about how the number of containers relates to the fraction of a litre in each?

Scaffolding learning

- Use actual containers or draw a diagram to help you visualise the problem.
- Use a number line.
- Start off with a small number of containers, e.g. 4 and a small number of whole litres in the big containers, e.g. 2. What fraction of juice will be in each of the 4 small containers?

Challenge

You double the amount of juice in each container and you end up with exactly 5 more full containers.
How many containers holding how much did you start with and end with?

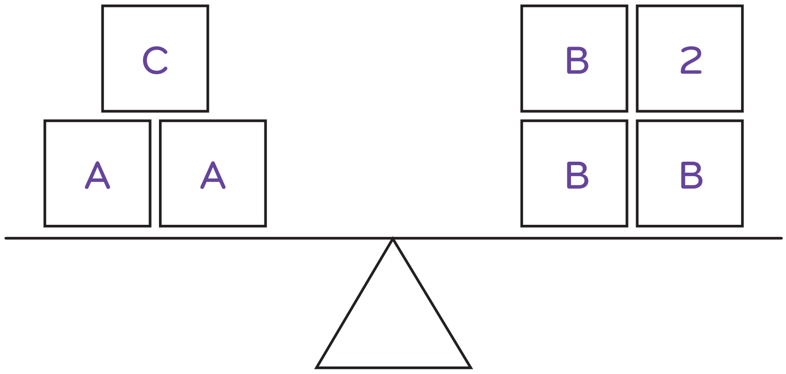
Name:

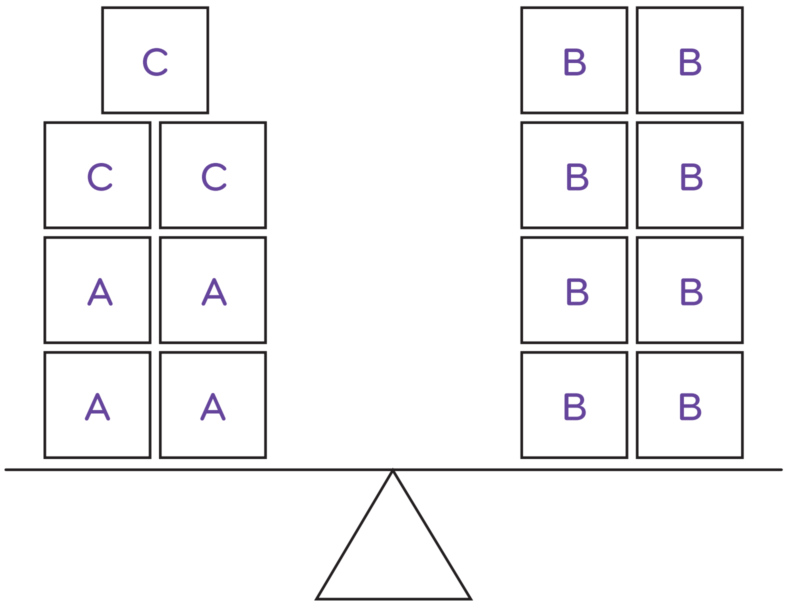
Pattern and Algebra

We Balance!

If these scales balance and the masses are whole numbers of kilograms, how heavy are boxes A, B, and C?

Get lots of possibilities.





We Balance!

Reasoning behind the task

This task engages students in some algebraic thinking, although they can solve the problem numerically. Some students will just try lots of different values for A, B, and C and hope that some work.

Others will use reasoning. For example, they might notice that the left side of the second balance is twice the left side of the first balance with an extra C, and use that information to simplify the problem. There are a number of correct solutions.

Curriculum coverage

- Pattern and Algebra
- Writing equations

Expectations

All	Most	Some
• Solve the problem at least one way through trial and improvement.	• Solve the problem a number of ways through a combination of trial and improvement, and reasoning. • Make a good attempt to explain the strategy used.	• Identify several different ways of solving the problem, using reasoning and correctly writes equations describing the balances. • Can describe the strategy used clearly, explain why C and B must be even amounts.

Key questions

- There are 8 Bs balancing 7 As and Cs. Does that mean that B must be greater than A and C? Explain.
- Do you think C can be an odd number? Explain.
- What about B? Explain.
- What equations could you write to describe the equalities?
- Suppose you just chose values for A and C and figured out B. Would that work?
- What if you decided that B and C were going to be the same value? Could you get a solution?

Scaffolding learning

- Try out several values for A, B and C and explore which balance the scales.
- Look at both balance scales – can you see any patterns which could help you simplify the problem.
- Write an equation to describe the what is happening.
- Solve the problem.

Challenge

Create a third balance that would work with one of the answers you got and see if it's possible to get a second possible answer to work with all three balances.