One Two

Series F

Rich Learning Tasks

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Problem Solving and Reasoning

One Two

Contents

Name:

Away from Home

Locations A, B, and C are all on the same road as Home.

Read and apply the following statements, replacing X with a value which is greater than 3:

- A is $\frac{3}{X}$ of the way to B from home.
- C is $\frac{2}{3}$ of the way to A from home.

Show where A, B, and C should be. You may estimate.

What fraction of the way to B is C? Can you explain your answer?

Apply different values for X. What do you notice?

Reasoning behind the task

Although this problem can be solved using multiplication of fractions by students who already have those skills, the problem can be solved using simpler fraction understandings. Even if students know how to multiply fractions, they will first have to recognise that this is a multiplication of fractions problem.
Students should come to realise that C is between home and A, and A is between C and B.
There was a deliberate choice to use $\frac{2}{3}$ as the second fraction so that students will think about the notion that $\frac{2}{3}$ of 3 __ths (e.g., 3 fourths, 3 fifths, etc.) is 2 __ths. Allowing students to select the denominator allows for a generalisation; students will see that the result is $\frac{2}{X}$, no matter what value is chosen for X.
That is because $\frac{2}{3} \times \frac{3}{X} = \frac{2}{X}$ for any chosen denominator.

Curriculum coverage

- Number
- Multiplying fractions

Expectations

All	Most	Some
• Place C, A, and B, in that order, on the number line for at least one choice of X. • Estimate the fraction of the way that C is from Home to B. • Notice that the distance between C and B are the same proportion for each choice of X.	• Accurately place C, A, and B, in that order, on the number line. • Correctly identify the fraction of the way that C is from Home to B. • Notice that C is $\frac{2}{X}$ of the way toward B.	• Accurately place C, A, and B, in that order, on the number line. • Realise that C is $\frac{2}{X}$ of the way toward B and can explain why. • Identify that they can use multiplication of fractions to find C, e.g. $\frac{3}{X} \times \frac{2}{3} = \frac{2}{X}$ • Know how to place B if A is placed first.

Away from Home

Key questions

- Is A or B closer to home? Why?
- Is C closer to A or to B? Explain your reasoning.
- Can A ever be more than halfway to B?
- If you placed A on the road, would it be possible to figure out where B was? How?
- How does your answer change if you change the value of X?

Scaffolding learning

- Consider your choice of value for X. Creating a fraction which you are familiar with will help you position the letters on the road more easily.
- Decide which letter to plot first. Plotting B is more challenging if A is plotted first. E.g. start by plotting B anywhere on the road. It is now possible to plot A using fraction estimating skills.
- Use the distance between Home and A to help identify where C should be plotted.
- Look carefully at the distance between Home and B. What fraction of this distance is C plotted at? Use your estimating skills, or exploring using multiplication of fractions to answer the question.
- Repeat using different values for X. Draw comparisons from your answers.

Challenge

Change the fractions $\frac{3}{X}$ and $\frac{2}{3}$ to another combination of fractions (e.g. $\frac{4}{X}$ and $\frac{1}{4}$) and repeat the activity.

Name:

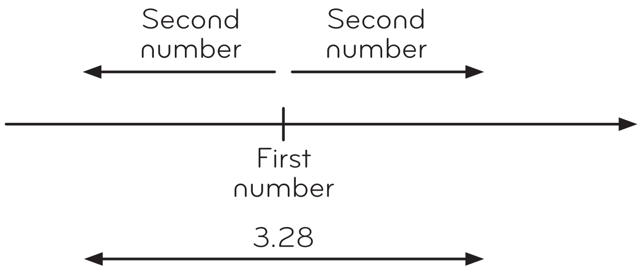
Add and Subtract

Add two numbers.
Subtract the same two numbers.
The answers are 3.28 apart.
What could your numbers have been?

Reasoning behind the task

Solving this problem will provide students with lots of practice adding and subtracting decimals, but will also lead to a surprising conclusion—the first number is irrelevant, but the second number has to be 1.64.

Some students will guess and test to solve the problem, but some might use a simple number line diagram and note that this is what has to happen:



Curriculum coverage

- Number
- Decimal calculations

Expectations

All	Most	Some
• Create one or more solutions using trial and improvement strategies. • Develop written methods for adding and subtracting decimals. • Recognise that the second number needs to be smaller than the first number.	• Create several solutions other than 0 and 3.28. • Accurately add and subtract decimals with tenths and hundredths. • Identify that the sum of the two numbers can be more than 3.28. • Describe the relationship between the two answers using a number line.	• Reason that any first number, whether a decimal hundredth or not, and a second number of 1.64 would satisfy the requirements. • Explain the solution to the problem using a number line to present their understanding.

Key questions

- Did both of the numbers have to be decimal hundredths or not?
- When you add the numbers, can the sum be more than 3.28?
- Could you have added and subtracted 5? Why or why not?
- Could you have added and subtracted more than 5 or less than 5?
- Could the first number be big? small? What are your choices?

Scaffolding learning

- If your first number was 1, what might you try for the second number? Would you be able to subtract that amount?
- Think about what mathematical written methods you could use to help you solve this problem. Explore using a number line to solve the problem.
- Choose your first number and place it on a number line. Think carefully about the difference between the two answers and the importance of the choice of second number. How can you create a difference of 3.28?

Challenge

Change the problem to: There are two decimal numbers. If you double the first one and add the second one, you get 4.93; if you double the second one and add the first one, you get 6.77. What might the numbers be?

Name:

How Much is 10,000?

Draw a picture that represents how much 10,000 is.
Explain why your picture makes that amount clear.

How Much is 10,000?

Number

Reasoning behind the task

The goal of this task is to help students make sense of a large number by relating it to more familiar numbers. To give a feel for how much 10,000 is, the student will need to show 10,000 as groups of, for example, 1000s, or 100s, or 10s. The focus should be on students' explanations of how the picture gives a true sense of the size of 10,000.

One possibility is that a student would create a set of 2 x 5 dots. They might replicate that set 10 times to make 100. That larger amount might be replicated 10 x 10 times to make 10,000.

Or a student might choose to show 20 piles of ten £50 notes. Or a student might choose to show the scale of a graph going up by 100 or 500 as far as 10,000.

Curriculum coverage

- Place value

Expectations

All	Most	Some
• Create a picture which represents 10,000. • Show a developing awareness of the relationship between numbers.	• Create a clear picture where 10,000 is related to another more familiar number in a multiplicative way (e.g.100 hundreds). • Describe the relationship of 10,000 to a more familiar number.	• Create a picture where 10,000 is related to another more familiar number in a multiplicative way (e.g.100 hundreds). • Explain the relationship of 10,000 to a more familiar number and recognise how that relationship helps give a sense of the size of 10,000.

How Much is 10,000?

Key questions

- How does your picture show how big 10,000 is?
- Do you think that your picture makes 10,000 look like a large or a small amount? Explain.
- What other number does your picture relate 10,000 to? Explain.

Scaffolding learning

- Think about what 10,000 means. Can you think of any real life examples when this quantity might be used? (e.g. money, objects, distance). Could this help you come up with an idea for a picture?
- Instead of drawing 10,000 individual shapes, are there any mathematical symbols you could use to show repetition?

Challenge

How many pieces of A4 paper would you need to show exactly 10,000 dots?

Name:

Multiplying Two by Two

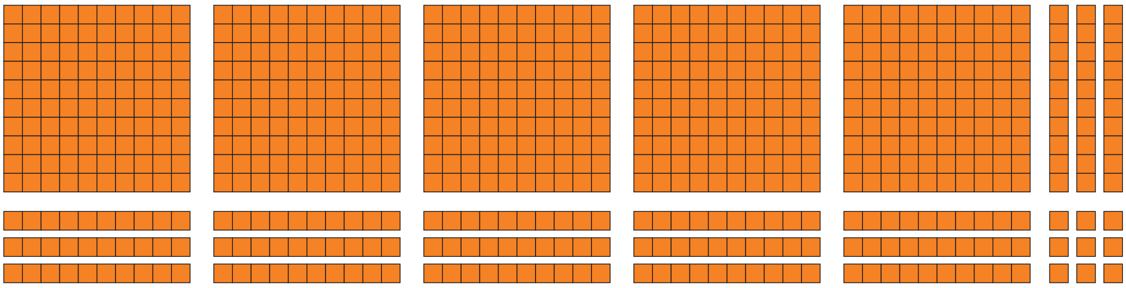
You model how to multiply two 2-digit numbers by using 32 base ten blocks.
What numbers could you be multiplying?
Show your working.
What is the product of these numbers?
Can you find more possibilities?

Multiplying Two by Two

Number

Reasoning behind the task

This task requires students to relate multiplication to area, an important connection, as well as to calculate products. In this case, providing the total number of blocks instead of giving a specific product, makes many results possible. For example, a student might choose 53 x 13 using a model like this:



Other choices might have been 26 x 22 or 97 x 11 or 71 x 31.

Notice that the number of blocks in a row multiplied by the number of blocks in a column equals 32.

Therefore, the sum of the digits of the numbers involved must also be factors of 32
e.g. 53 x 13 → (5 + 3=8) x (1 + 3=4) 8 x 4=32

Curriculum coverage

- Number
- Multiplication
- Proportional Relationships

Expectations

All	Most	Some
• Identify one or more possible pairs of numbers where the model for multiplication could use 32 blocks.	• Identify possible pairs of numbers where the model for multiplication could use 32 blocks. • Find the products. • Use an area of a rectangle model.	• Recognise that the sum of the digits of the numbers involved is a factor of 32; using this method, identify possible pairs of numbers where the model for multiplication could use 32 blocks. • Explain the relationship between multiplication and area using a rectangle.

Multiplying Two by Two

Key questions

- How were your blocks arranged?
- What meaning of multiplication does your model depend on?
- Could you have looked at the numbers and predicted you would need 32 blocks to show the multiplication model? How?

Scaffolding learning

- Explore different quantities of base ten blocks (100, 10, 1).
- Think about how you might arrange base ten blocks to show e.g. 2 x 12. Could you arrange them in a rectangle? Explore how many base ten blocks you would need for 2 x 12 using the least amount possible (two tens and four units).
- Make a rectangle using 32 blocks using trial and improvement.
- Explore how you could use the factors of 32 to help you.

Challenge

Identify two numbers where the model for the product requires the use of 45 base ten blocks.

Name:

Moving Around

Start at the bottom left corner of the grid.

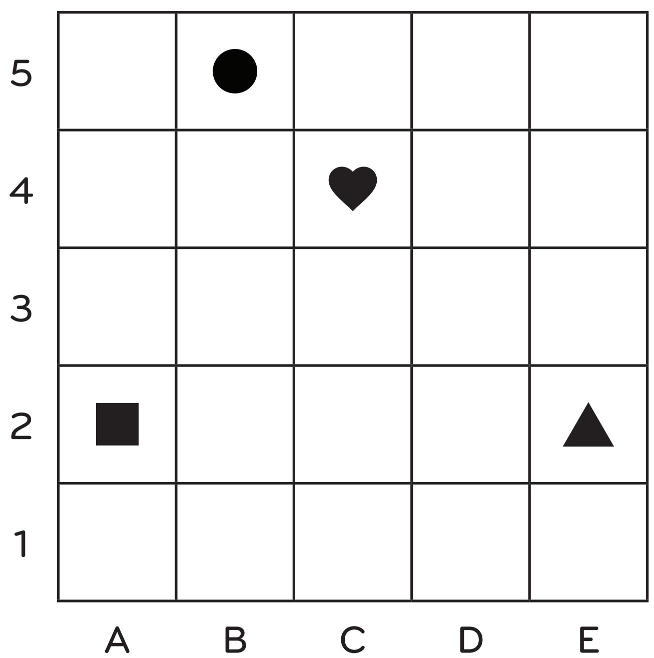
Describe a path of single-square moves, right, left, up or down, you would have to make to collect all four objects. How many moves long is your path?

Now place two more objects on the grid so that it takes a total of 15 single-square moves to collect all 6 objects, still starting at the bottom left corner.

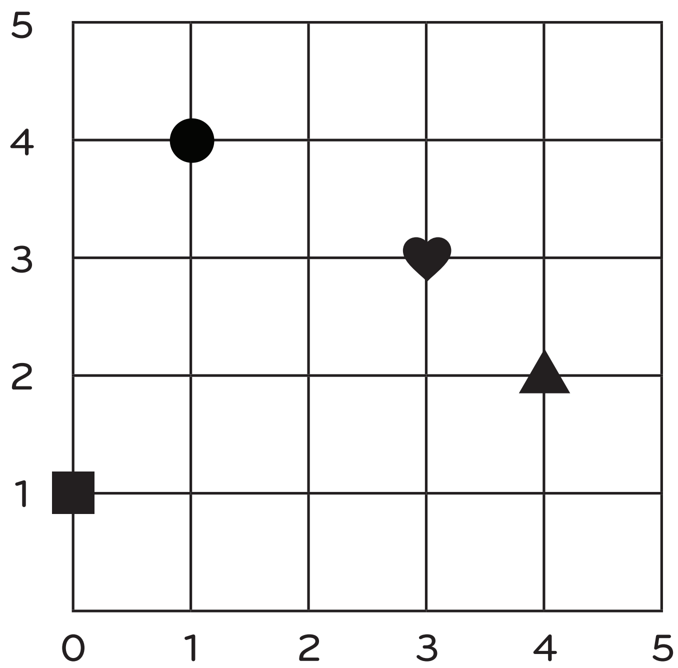
Describe the 15-move path, starting at the bottom left corner.

Use either Grid 1 or Grid 2.

GRID 1



GRID 2



Reasoning behind the task

Defining location on a coordinate grid is essential background for later grades in math. There are two kinds of grids provided in this problem—Grid 1, a "city grid" where locations are based on positions in cells and Grid 2, a Cartesian grid system where locations are based on points. Both grids allow for students to describe location and movement.

A problem element is added by asking students to move exactly 15 moves to collect all 6 objects. For example, on Grid 1, students could add objects in cells B2 and D5 and move 1 space up to A2, 1 space right to B2, 3 spaces up to B5, 1 space right and 1 space down to C4, 2 spaces right and 2 spaces down to E2, and 3 spaces up and 1 space left to D5. On Grid 2, students could add objects at (2, 2) and (2, 5) and move 1 space up to
(0, 1), 3 spaces up and 1 space right to (1, 4), 2 spaces right and 1 space down to (3, 3), 1 space right and 1 space down to (4, 2) and 3 spaces up and 2 spaces left to (2, 5).

Students might explore how paths of different lengths might be used depending on the sequence in which the objects are collected. It is important for students to know that there are generally many paths to get from one point to another. For example, to get from (2, 2) to (3, 3) you could move right first and then up or the other way around.

Curriculum coverage

- Geometry
- Coordinates
- Translations

Expectations

All	Most	Some
• Identify a path to collect all four objects. • Place two more objects on the grid and explore possible paths to collect all six objects.	• Describe a path to collect all four objects initially on the grid. • Place two more objects on the grid and correctly describe a 15-square path to collect all 6 objects.	• Place two more objects on the grid and correctly describe a 15-square path to collect all 6 objects. • Generalise how locations work on a grid and can describe the number of moves given two locations without seeing the grid. • Explain the value of a grid system when asked.

Key questions

- Is there more than one way to get from the square to the circle? Do you move the same total number of spaces using different paths?
- What different length paths could you use to collect the 4 objects on the grid? What is the shortest length?
- Could you use the location of two objects, without seeing the grid, and figure out how long the path from one object to the other is?
- How does using a grid system help you to locate the objects?
- Does it matter where you add the two further objects?

Scaffolding learning

- Identify differences between the two grids. Choose which grid you are going to use.
- Practise reading and recording grid references, e.g. A2, (0,1).
- Select and record your path, e.g. up one space to A2. Continue until you have collected all the objects on the grid. Explore alternative paths; are they shorter or longer?
- Do you end up in the same place if you move 1 right and then 1 up or 1 up and then 1 right? Do you end up in the same place if you move 1 right and 2 up or 2 right and 1 up?
- Add two more objects to your grid and investigate a path which takes a total of 15 single square moves. Think about your choice of object positioning.

Challenge

Create a larger grid and place 4 objects so that the total length of the path used to pick up all 4 objects would be 25 moves.

Name:

Geometry

Two Points

The two dark dots shown could be part of one or two edges or could be vertices of the shape you see after you reflect a triangle using a mirror. If you reflect a triangle using a mirror, the two dark dots shown could either be vertices, or part of one or two edges of the reflected shape.

Draw the original triangle, the reflection line, and the reflected shape that the dots are part of.

Try different possibilities.

Two Points

Reasoning behind the task

This task encourages students to visualise the effects of a reflection. Because only a small bit of the reflected image is shown, and none of the original image or reflection line is shown, and because students can choose to use those dots as any part of the reflection image, there are many, many possible solutions. The reflection line could actually go through the dots, or be parallel to the line that joins them, or could intersect the line that joins them.

The students are not told that the image they draw needs to be a triangle. Students who are comfortable with reflections should realise this. Other students may need more support. Some students will assume that the reflection line must be parallel to the segment formed by the two dots, but this is not necessarily the case.

Curriculum coverage

- Geometry
- Reflections

Expectations

All	Most	Some
• Identify that the reflected shape needs to be a triangle. • Explore possible pre-image triangles, reflection lines, and image triangles through the two dots.	• Correctly draw a pre-image triangle, a reflection line, and an image triangle through the two dots in one or more different ways. • Attempt to explain how they know the image is correctly placed. • Recognise how the orientation of a shape changes as a result of a reflection.	• Correctly draw a pre-image triangle, a reflection line, and an image triangle through the two dots in number of ways. • Explain clearly how they know the image is correctly placed. • Recognise how the orientation of a shape changes as a result of a reflection. • Realise that the reflection line could have gone through, been parallel to, or have intersected the line formed by the two dots.

Two Points

Key questions

- What could the reflected shape look like?
- Were the points on the original triangle that were reflected onto the two black dots arranged in exactly the same way as the two black dots?
- Was the reflection line parallel to both of the dark dots? Did it have to be?
- Was the original triangle on the left or the right of the two reflected dots? Could it have been either?

Scaffolding learning

- Think about what possible shapes the reflected shape could have been. Why?
- Explore where the reflection line might be. Test different positions and consider what part of the reflected shape the two dots might be (edges or vertices).
- Draw a shape incorporating the dots as vertices. Choose a reflection line and use a mirror to identify the original shape. Repeat with the dots as edges. What if the dots were on two different edges of the reflected shape?

Challenge

Investigate the original shape, reflection line, and reflection image if three dots were shown.

Three Cuboids

A short, medium, and tall cuboid, together, have a total volume of 80 cubic units.

What might be the length, width, and height of each?

Think of a number of possibilities.

Three Cuboids

Reasoning behind the task

Students will become familiar with determining the volume of a cuboid using information about its height and the area of its base (volume = length x width x height). Rather than giving the volume of a single cuboid, it is more interesting to have a total volume for three cuboids. Students are asked to use a tall, medium, and short cuboid to require them to vary heights; they can choose to vary the areas of the bases or not. For example, if the area of the base were 4 square units, there could be three cuboids with a base of 4 square units, one with a height of 2 units, one with a height of 6 units, and one with a height of 12 units.

Or students could use a 1 x 1 x 2 cuboid (with a height of 2), a 2 x 3 x 5 cuboid (with a height of 5), and a 24 x 1 x cuboid (with a height of 24).

Curriculum coverage

- Measurement
- Volume of a cuboid

Expectations

All	Most	Some
• Explore possible volumes for cuboids • Correctly identify the volume for each cuboid by counting cubes.	• Create one or more sets of three cuboids with the appropriate heights and the correct total volume. • Correctly identify the volume of each cuboid by using a formula, formally or informally.	• Create several possible sets of three cuboids with the appropriate heights and the correct total volume. • Identify the volume of each cuboid by using a formula, formally or informally. • Notice that the bases could all be the same in area, or not.

Three Cuboids

Key questions

- What information do you need to find the volume of a cuboid?
- Is it possible for cuboids with different heights to have the same volumes? How could that happen?
- Do the bases of the three cuboids have to be the same? Could they be?
- Could any of the heights of the cuboids be more than 20 units? Why or why not?
- Could any of the cuboids be more than half the total volume? Why or why not?

Scaffolding learning

- Find out the formula for finding the volume of a cuboid.
- Estimate possible volumes for the small, medium and large cuboid.
- Use trial and improvement strategies to explore possible combinations of volumes to make 80^3.
- Try to use your knowledge of multiplication, factors and division to help you solve the problem.

Challenge

Create three prisms where the middle volume is double the first, the third volume is double the second, and the combined volume is 84 cubic units?

Name:

Measurement

Big and Little Rectangles

The area of one rectangle is $2\frac{1}{2}$ times as big as the area of another.
What could the original and new lengths and widths be?
Show at least three possibilities starting with the same small rectangle.
Can you do the same thing if you start with a different small rectangle?

Big and Little Rectangles

Reasoning behind the task

Creating a rectangle with an area $2\frac{1}{2}$ times as big as another's engages students in a number of mathematical concepts. It might involve using the formula for areas of rectangles. It certainly involves students in using multiplicative reasoning, recognising that $2\frac{1}{2}$ of something is two of it and another half of it. Students might realise they could leave the length and multiply the width by $2\frac{1}{2}$ or leave the width and multiply the length by $2\frac{1}{2}$, or they might change both measures—e.g., multiply the length by 5 and take half of the width. Since students are asked for at least three possibilities, they will have to use at least one example where both dimensions are changed. Students are most likely use trial and improvement strategies, or some may apply the reasoning that $2\frac{1}{2} = \frac{5}{2}$.

Students could use a rectangle with one even dimension so that it could be easily cut in half and simply copy the rectangle and another half of it, put the pieces together, and then identify the dimensions.

Asking if the first size of the rectangle matters gives students an opportunity to generalise.

Curriculum coverage

- Measurement
- Area of rectangles
- Proportion

Expectations

All	Most	Some
• Create at least one rectangle with $2\frac{1}{2}$ times the area of an initial rectangle; possibly use a rectangle with one even dimension to cut in half and copy one and a half times.	• Create three different rectangles with $2\frac{1}{2}$ times the area of an initial rectangle. • Show an understanding that either the length or width or both could change.	• Create three different rectangles with $2\frac{1}{2}$ times the area of an initial rectangle. • Handle measurements involving decimals. • Explain why either the length, width or both can change.

Big and Little Rectangles

Key questions

- What does $2\frac{1}{2}$ times something mean?
- Could you change just the length?
- If the length was doubled, how would the width change?
- If the length was not changed, how would the width change?
- Is it possible that the rectangle's length could change from 5 cm to 15 cm? Explain.

Scaffolding learning

- Think about how to find the area of a rectangle. What is the formula?
- Choose measurements for a rectangle and find its area. Use these measurements to find the area of a rectangle two and half times larger.
- Investigate what happens to the area if you multiply just the width of the smaller rectangle by $2\frac{1}{2}$.
- Investigate what happens to the area if you multiply just the length of the smaller rectangle by $2\frac{1}{2}$.
- Continue to investigate until you find at least three possible dimensions for rectangles with an area two and a half times larger than the original.

Challenge

Find the measurements for a rectangle that is three times as long as it is wide, with an area that is the same as its perimeter.

Name:

Related Patterns

Pattern 1: 3, 6, 9, 12, ...
Pattern 2: 11, 21, 31, 41, ...

Numbers in patterns are called terms. The terms in the patterns are called "matching" if they are both at the same position in the pattern, e.g. 3 and 11 are matching terms because they are both in position one.

Draw a picture that shows that every term in pattern 2 is one more than $3\frac{1}{3}$ lots of the matching term in pattern 1.

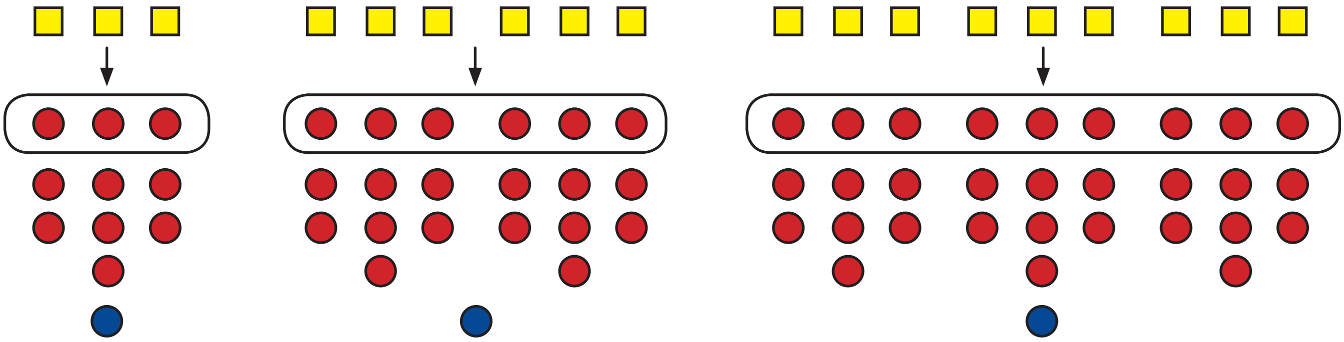
Explain how your picture shows this.

Reasoning behind the task

Linear patterns—number patterns with a constant increase or decrease—are fundamental in many aspects of math. What is particularly interesting about them is that in each case, you can predict the term value in one pattern using a simple relationship to the term value in the other. For example, a very simple example is 2, 4, 6, 8, 10, ... compared to 4, 8, 12, 16, 20, ... You can determine any term value in the second pattern by doubling the corresponding term value in the first one; similarly, you can determine any term value in the first pattern by halving the corresponding term in the second one.

The patterns used in this task are slightly more complex, but not too complex. Since the first pattern is a simple 3-times table and the second pattern is the 10-times table with 1 added, the suggested rule makes sense. What will be interesting is to see how a student might show it visually.

One possibility is to use a pictograph; another possibility is to use a pattern of figures like this one.



Curriculum coverage

- Patterns and algebra
- Linear sequences

Expectations

All	Most	Some
• Draw a picture that shows some corresponding values in the two patterns.	• Draw a picture that shows the $3\frac{1}{3}$ times relationship between corresponding values in the two patterns. • Demonstrate an understanding for why this pattern will have to continue.	• Draw a clear picture that shows the $3\frac{1}{3}$ times relationship between corresponding values in the two patterns. • Understand why the pattern will have to continue. • Describe how the relationship works in reverse, going from pattern 2 to pattern 1.

Related Patterns

Key questions

- What does $3\frac{1}{3}$ times a number mean?
- What would happen if you multiplied 3, 6, and 9 by $3\frac{1}{3}$?
- How do you know this will continue even for terms much later in the pattern?
- How would you get the value of a term in pattern 1 if you knew the value in pattern 2 for the term in the same position? [You would subtract 1 and find $\frac{3}{10}$]

Scaffolding learning

- Create a visual representation of pattern one; choose something easy to repeat, e.g. a circle or a dot.
- Discuss what $3\frac{1}{3}$ times a number means and looks like? Represent this mathematically using repeating symbols.
- Think about whether the pattern will continue in the same way or not.
- Using your picture to help you, think about the relationship working in reverse, going from pattern 2 to pattern 1.

Challenge

Find a rule to get from a term in the pattern 5, 9, 13, 17, 21, ... to the term in the same position in the pattern 5, 8, 11, 14, 17, ...

Name:

Spin Red, Green, and Blue

On a spinner, you are twice as likely to get red as blue, and half as likely to get blue as green.

What could the spinner look like? Make it using a spinner and red, blue and green labels.

Test to see if you are right by spinning 10 times to see what happens.

Is there more than one possibility? Explain.

Spin Red, Green, and Blue

Reasoning behind the task

Although this task relates to probability, students are not required to actually state probabilities. In fact, all they need to be able to do is to understand fractions of a whole to solve the task. There was a deliberate choice to state that the red is twice as likely as blue, but to state that blue is half as likely as green (rather than saying green is twice as likely as blue); this is to help students make the connection that $a = 2b$ means that $b = \frac{1}{2}a$.

Curriculum coverage

- Data
- Probability
- Fractions
- Algebra

Expectations

All	Most	Some
• Create at least one correct spinner and test it appropriately.	• Create more than one correct spinner and test it appropriately. • Create an alternative spinner or explain why alternatives are possible. • Explain why the minimum number of sections is 5.	• Create an alternative spinner or explain why alternatives are possible. • Explain why the minimum number of sections is 5. • Explain why the experimental results might not match what was expected.

Spin Red, Green, and Blue

Key questions

- Which section is bigger—red or blue? Why?
- Which section is bigger—green or blue? Why?
- Could the blue section be $\frac{1}{4}$ of the spinner? Why or why not? Does it matter where you position the colours?
- How do the red and green areas compare? Why did that happen?
- What is the least number of sections that the spinner could have? Why?
- What other numbers of sections, in total, could the spinner have?
- Should you redraw your spinner if, when you spin 10 times, it doesn't work out the way you expected?

Scaffolding learning

- Use a visual resource to represent the conditions, e.g. coloured counters, pictures or letters.
- Think about ratio and proportion and the minimum sections required on the spinner for the conditions to apply. How could you include more sections on your spinner?
- Make your spinner and test it by spinning it ten times.

Challenge

Can you create your own colour relationship conditions and make a corresponding spinner?