Mathletics

50 – (45 ÷ 9) + 8=20÷4
(60 – 8) × 2 + (16 ÷ 4) – 32
30 ÷ (4 + 11)(△ × 12) × 5 = 120

Patterns and Algebra

My name

Series G – Patterns and Algebra

Contents

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Patterns and functions – recursive number sequences

A number pattern is a sequence or list of numbers that is formed according to a rule.

Number patterns can use any of the four operations (+, –, ×, ÷) or a combination of these.

There are 2 different types of rules that we can use to continue a number pattern:

1 A recursive rule – find the next number by doing something to the number before it.

2 A function rule – predict any number by applying the rule to the position of the number.

Here is an example of a number sequence with a recursive rule.

The rule is add 8 to the previous number, starting with 5.

| 5 | 13 | 21 | 29 | 37 |

+ 8   + 8   + 8   + 8

---
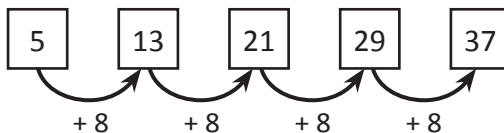
1 Figure out the missing numbers in each pattern and write the rule:

a   9   18   [ ]   36   45   [ ]

Rule _____

b   10   [ ]   [ ]   37   46   [ ]

Rule _____

c   125   100   [ ]   50   25   [ ]

Rule _____

d   49   42   [ ]   28   21   [ ]

Rule _____

e   7   13   [ ]   25   31   [ ]

Rule _____

f   3   [ ]   17   24   31   [ ]

Rule _____

---

2 What do you notice about the patterns a and b in Question 1?

_____

---

3 Complete these grid patterns. Look closely at numbers in the grid and follow the pattern going vertically and horizontally:

a

16			19
26			
		39	
46			

b

10			25
	24		
	33		
37			52

c

			45
	35		
34		52	61

Patterns and functions – recursive number sequences

4 **Complete these sequences according to the recursive rule:**

a | Start at 3 and add 7 | 3 → ☐ → ☐ → ☐ → ☐

b | Start at 125 and subtract 5 | 125 → ☐ → ☐ → ☐ → ☐

c | Start at 68 and add 20 | 68 → ☐ → ☐ → ☐ → ☐

5 **Complete these decimal number sequences according to the recursive rule:**

a | Start at 2.5 and add 0.5 | 2.5 → ☐ → ☐ → ☐ → ☐

b | Start at 25 and subtract 0.5 | 25 → ☐ → ☐ → ☐ → ☐

c | Start at 30 and add 2.5 | 30 → ☐ → ☐ → ☐ → ☐

6 **Complete the following number patterns and write the rule as two operations in the diamond shapes and describe it underneath.**

a 1 ◁ × __ + __ ▷ 3 ◁ × __ + __ ▷ 7 ◁ × __ + __ ▷ 15

The rule is _____

b 2 ◁ × __ − __ ▷ 3 ◁ × __ − __ ▷ 6 ◁ × __ − __ ▷ 15

The rule is _____

7 **Work out where each pattern started to go wrong in these single operation patterns and circle the incorrect numbers. Hint: The first two numbers in both are correct.**

a
| 78 | 100 | 122 | 144 | 166 | 188 | 211 | 222 | 233 |

The rule is _____

b
| 500 | 466 | 432 | 398 | 364 | 330 | 298 | 266 | 230 |

The rule is _____

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Patterns and Algebra

Patterns and functions – function number sequences

There are two different types of rules that we can apply to find out more about a sequence:

1 A recursive rule – gives the next number by applying a rule to the number before it

2 A function rule – predict any number by applying a rule to the position of the number

So far we have practised the recursive rule to work out the next number in a sequence.
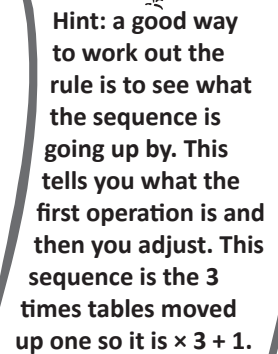
Now we will apply the function rule to this problem:

How can we find out the 20th number in this sequence without writing out all of the numbers?

To use the function rule we:

• Use a table like this one below.

• Write each number of the sequence in position.

• Work out the rule, which is the relationship between the position of a number and the number in the pattern.

• Use the rule to work out the 20th number in the sequence.

Position of number	1	2	3	4	5	20
Rule	× 3 + 1	× 3 + 1	× 3 + 1	× 3 + 1	× 3 + 1	× 3 + 1
Number sequence	4	7	10	13	16	61

> Hint: a good way to work out the rule is to see what the sequence is going up by. This tells you what the first operation is and then you adjust. This sequence is the 3 times tables moved up one so it is × 3 + 1.

1 **In each table, find the rule and write it in the middle row. Then apply the rule to position 20.**

a

Position of number	1	2	3	4	5	20
Rule						
Number sequence	6	11	16	21	26	

b

Position of number	1	2	3	4	5	20
Rule						
Number sequence	5	7	9	11	13	

c

Position of number	1	2	3	4	5	20
Rule						
Number sequence	8	17	26	35	44	
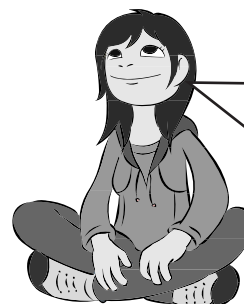
> **HINT: All of these function rules consist of 2 operations: × and then + or −.**

Patterns and functions – function number sequences

2   Here is part of a number sequence. Write these numbers in the table provided. This will help you to answer the questions below:

8	11	14	17	20

Position of number	1	2	3	4	5	20
Rule						
Number sequence						

....

3   Circle true or false for each of the following:

a   The number in the 6th position is 24   true / false

b   32 is in this sequence   true / false

c   The number in the 20th position is 65   true / false

d   The number in the 100th position is 305   true / false

....

4   Here is another number sequence but this time 4 of these numbers do not belong. Given the function rule and the first 2 numbers, use the table below to work out how this sequence should go, then cross out the numbers that do not belong:

15	23	31	36	47	59	63	74	79	97

Position of number	1	2	3	4	5	6	7	8	9	10
Rule	×8+7	×8+7	×8+7	×8+7	×8+7	×8+7	×8+7	×8+7	×8+7	×8+7
Number sequence										

....

5   Unscramble the sequence according to this function rule: × 9 − 6.

a   Again, use the table below to work out how this sequence should go and cross out numbers that do not belong:

9   78   48   23   3   60   30   12   66   84   57   21   63   36   39   33   27   75   18

Position of number	1	2	3	4	5	6	7	8	9	10
Rule										
Number sequence										

b   What will be the number in position 50? _____

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Patterns and Algebra

Patterns and functions – function shape patterns

When you are investigating geometric patterns, look closely at the position of each shape and think about how it is changing each time.

How many matchsticks are needed for the first shape?

How many more are needed for the next shape?

1 Complete the table for each sequence of matchstick shapes. Use the function rule for finding the number of matchsticks needed for each shape including the 50th shape:
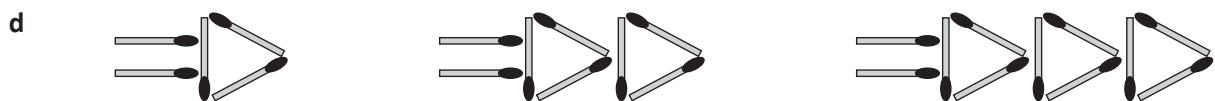
a
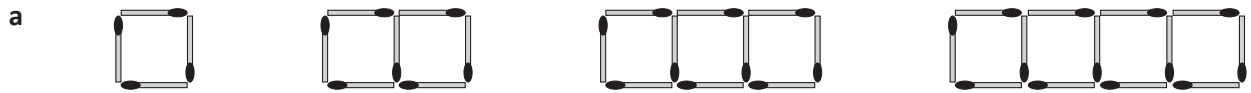
Shape number	1	2	3	4	5	6	7	8	9	10	50
Number of matchsticks	4	7	10	13	16						
Function rule	Number of matchsticks = Shape number × _____ + 1										

b
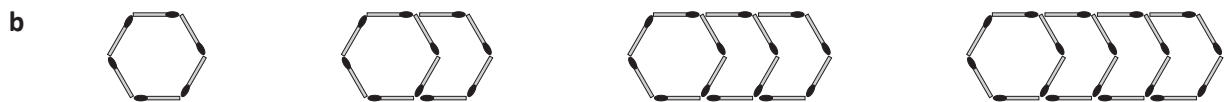
Shape number	1	2	3	4	5	6	7	8	9	10	50
Number of matchsticks	6	10	14	18	22						
Function rule	Number of matchsticks = Shape number × _____ + _____										

c

Shape number	1	2	3	4	5	6	7	8	9	10	50
Number of matchsticks	3	5	7	9	11						
Function rule	Number of matchsticks = Shape number × _____ + _____										

d

Shape number	1	2	3	4	5	6	7	8	9	10	50
Number of matchsticks	5	8	11	14	17						
Function rule	Number of matchsticks = Shape number × _____ + _____										

Patterns and functions – function shape patterns

2 **Gia started to make a sequence out of star and pentagon blocks and recorded her findings in the table as she went. She had to stop when she ran out of pentagons. This is where she got up to:**


a Help Gia continue investigating this sequence by using the table below:

Shape number	1	2	3	4	5	6	7	8	9	10	15
Number of stars	1	2	3								
Number of pentagons	0	1	2								
Rule for stars	Number of stars = Number of pentagons + 1										
Rule for pentagons	Number of pentagons = Number of stars − 1										

b How many stars are in the 10th shape?

c How many pentagons are there in the 15th shape?

....

3 **Tyson also made a sequence out of pattern blocks but stopped after the first 3 shapes and decided to continue investigating by using the table.**
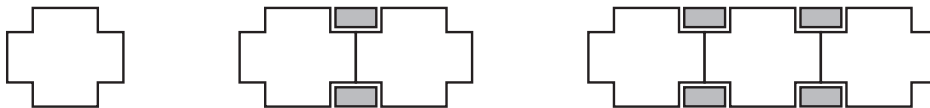
Shape number	1	2	3	4	5	6	7	8	9	10
Number of crosses	1	2	3							
Number of rectangles	0	2	4							
Rule for crosses	Number of crosses = (2 + number of rectangles) ÷ 2									
Rule for rectangles	Number of rectangles = (2 × number of crosses) − 2									

a How many rectangles will there be in the 12th shape?

b Josie made this shape following Tyson's sequence.
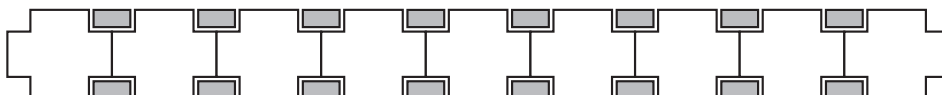
What is the position of this shape? _____

How do you know?

Patterns and Algebra

# Patterns and functions – function machines and function tables

Remember function machines? Numbers go in, have the rule applied, and come out again.

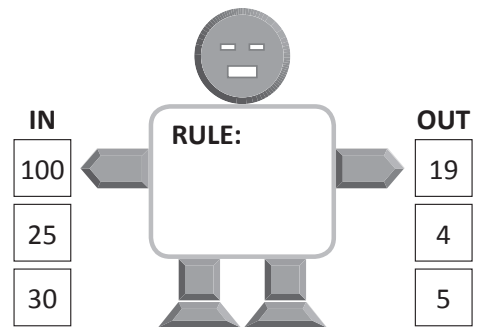The rule for this function machine is **multiply by 6**.

IN	RULE: × 6	OUT
5		30
9		54
12		72
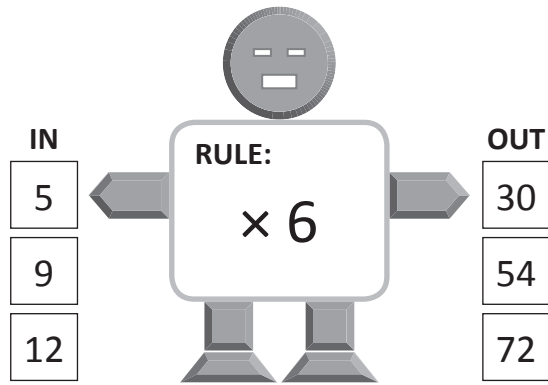
1 Look carefully at the numbers going *in* these function machines and the numbers coming *out*. What 2 rules are they following each time?

a

IN	RULE:	OUT
8		59
5		38
9		66

b

IN	RULE:	OUT
100		19
25		4
30		5
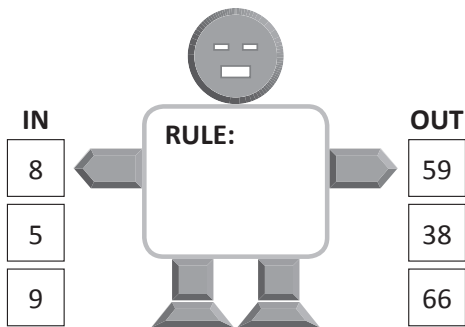
The function machines showed us that when a number goes in, it comes out changed by the rule or the function. Function tables are the same idea – the number goes **in** the rule and the number that comes **out** is written in the table. The rule goes at the top:

Rule: ÷ 2 + 6								
IN	10	24	50	70	48	90	100	80
OUT	11	18	31	41	30	51	56	46

2 Complete these function tables according to the rule:

a

Rule: × 8 + 1								
IN	8	2	3	5	7	9	4	6
OUT	65							

b

Rule: × 5 − 4								
IN	6	9	3	4	7	11	20	8
OUT	26							

Patterns and functions – real life functions

So far we have seen that functions are relationships between numbers.

These numbers are attached to real life situations everywhere you look. It is possible to create a function table to show the relationship between many things, for example:

- Your high score Live Mathletics depends on how often you practise mental arithmetic.
- The distance that you run depends on how long you run.
- The amount that you can save depends on how much you earn.
- The amount of US dollars you get when you travel to Los Angeles depends on the exchange rate.

There are many, many more examples. Can you think of any?

1 **Complete the function tables for these real life scenarios:**

a A pool which fills at a rate of 4 litres every minute.

Rule: Number of minutes × 4 = Amount of litres							
Minutes	5	10	15	20	25	30	35	40
Litres	20	40	60	80				
How full is it after one hour?							

b Maya downloads 5 songs a day onto her MP3 player.

Rule: Number of days × _____ = Amount of songs							
Days	1	2	3	4	5	6	7	8
Songs	5	10	15	20				
How many songs would she have downloaded after 30 days?							

c A car is travelling at a speed of 50 km/h.

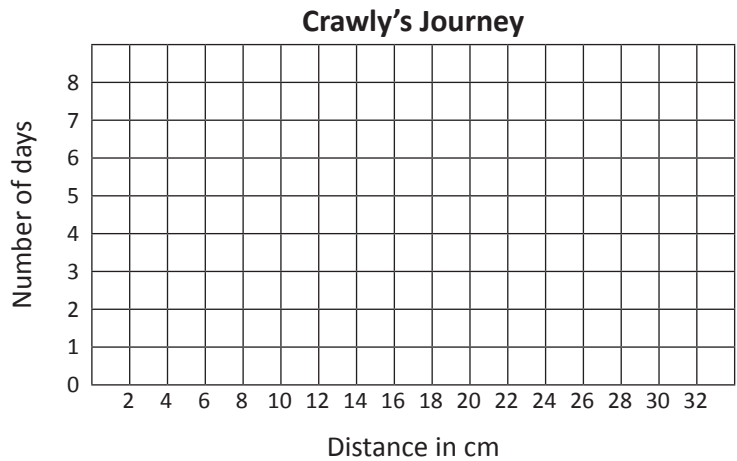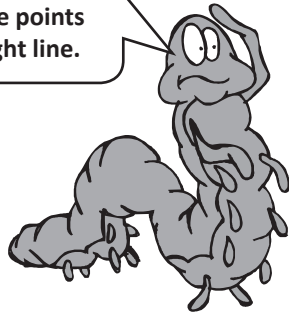| Rule: Number of hours × _____ = Amount of km travelled							
Hours	1	2	3	4	5	6	7	8
Km travelled	50	100	150	200				
How long would it take to travel 800 km?							

We can show these relationships on a graph. On the right is a graph of the function table in question **c**. This is known as a travel graph and shows the relationship between time and distance. Next, we will look at some examples of graphing functions.

Leah's Journey

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Patterns and Algebra

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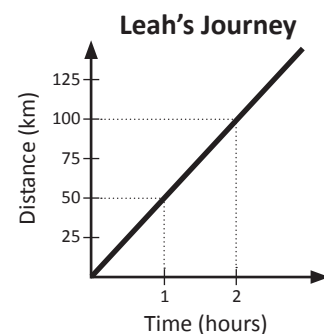
Patterns and functions – real life functions

2   **Crawly the caterpillar crawls 4 centimetres per day.**

> This is the graph of my journey shown in the function table. Plot the points and then join the points with a straight line.

Crawly's Journey

(Graph: vertical axis "Number of days" 0–8, horizontal axis "Distance in cm" 2–32)

a   Complete the table to show how far he gets in 8 days.

b   Write a rule for working out the distance if you know the number of days.

Rule:								
Days	1	2	3	4	5	6	7	8
Distance								

---

3   **During the day, Crawly's friend Creepy, crawls 5 cm up a garden wall. At night when he falls asleep, he slides 2 cm back down the wall.**

Creepy's Journey


(Graph: vertical axis "Number of days" 0–8, horizontal axis "Distance in cm" 2–32)
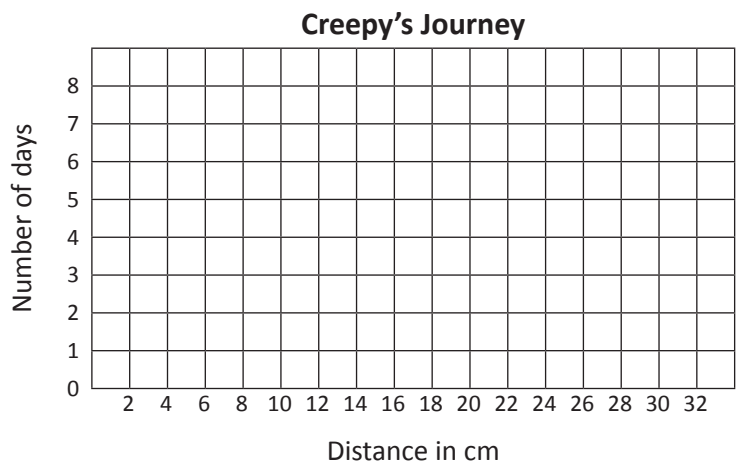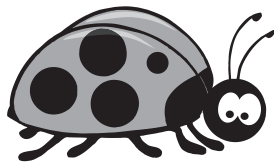
a   Complete the table below to show how far he gets in 8 days.

Rule:								
Days	1	2	3	4	5	6	7	8
Distance								

b   Write a rule for working out the distance if you know the number of days. Think about the total distance Creepy covers in 24 hours.

c   Plot the points on the graph above (just like the one in Question 2), then compare the graphs. How are they different?

_____

Patterns and functions – real life functions

4 **Julie is planning her birthday party and is planning how much food and drink she needs for her guests. She has sent out 15 invitations.**
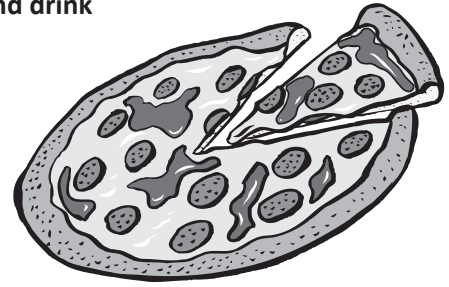
a Complete the table to show how much pizza is needed for different numbers of guests. She has based this table on the estimation that one guest would eat 3 slices of pizza.

b Write a rule in the table for working out the slices of pizzas needed, if you know the number of guests.

Rule:								
Number of guests	1	2	3	4	5	6	7	8
Slices of pizza	3							

c Graph this data by plotting the points from the table:

Do not join the points because the data is about whole slices of pizza – not parts of slices. Also you cannot have part of a person, the data is about single people.

d How many slices are needed for 11 people?

Pizza catering

e How did you work this out?

f How could the graph help you?
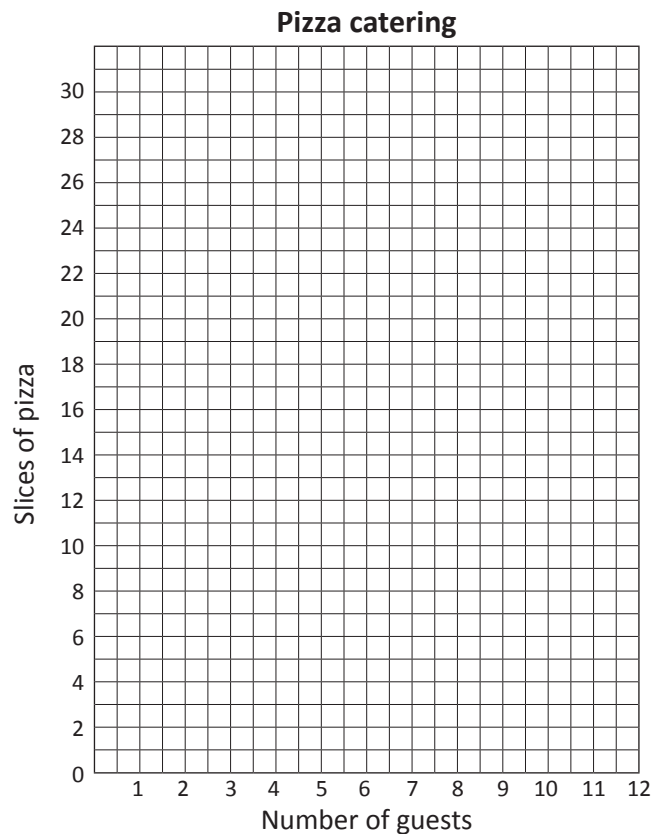
g 10 people confirmed they were coming to the party. How many pizzas will Julie need to buy if each pizza has 12 slices? Will there be any leftovers? Show your working.

Patterns and Algebra

Function tables   apply

You and your partner need 2 dice, a pencil and this page.

What to do

1 Each player writes their initials at the top of each column in the scoring tables.
2 For each round, roll the dice for ✹ and ☺.
3 Use the value for ✹ and ☺ in the rule.
4 Each player writes the answer in the scoring table which becomes their running score.
5 Players add their scores to the previous score.
6 The winner is the player with the highest score at the end of the round. The overall winner is the player who wins the most points after 3 rounds.

For example If I roll the dice and get 4 for ✹ and 6 for ☺ and I am working with (2 × ✹) + ☺, I would calculate (2 × 4) + 6 and my answer would be 14. So I would write 14 in the first row of the table. The next answer I get I add to 14 and so on until the end of the table.

Round 1 $(2 \times ✹) + ☺$

Round 2 $(3 \times ✹) + ☺$

Round 3 $(6 \times ✹) - (2 \times ☺)$

Total	

Total	

Total	

What to do next

Make up your own scoring table where extra points are given for certain answers.

You could also decide on a 'killer number'. This number means you wipe out all your points.

Getting ready

A very popular wedding reception venue has a strict policy in the way they put the tables and chairs together. Below is a bird's eye view of this arrangement. They must only be arranged in this sequence to allow room for their famous ice sculptures in the centre of each table arrangement.

What to do

Table and Chair Arrangement 1	Table and Chair Arrangement 2	Table and Chair Arrangement 3
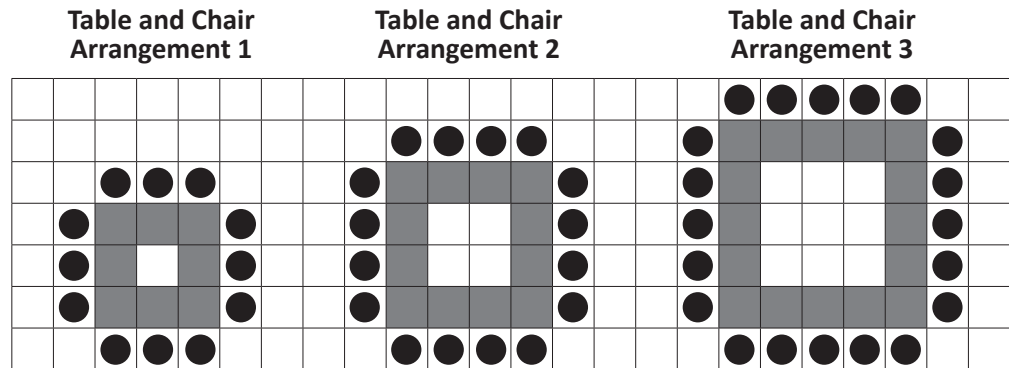
Look carefully at the diagram of the floor plan above.

a Complete the table below.

b Write the rule in the table for the number of tables needed if you know the table and chair arrangement number.

c Write the rule in the table for the number of chairs needed if you know the table and chair arrangement number.

d Draw what Table and Chair Arrangement 4 would look like in the grid at the bottom of this page.

Table and Chair Arrangement	1	2	3	4	5	6	7	8	9	10
Tables	8	12	16							
Chairs	12	16	20	24	28	32	36	40	44	48
Rule for tables										
Rule for chairs										

Table and Chair Arrangement 4

Patterns and Algebra

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What to do next
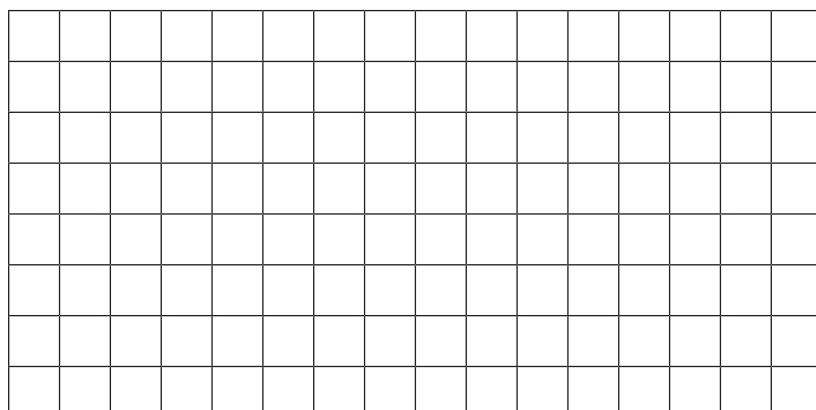
The latest Bridezilla to hire out the "I Do" venue, wants to know how many guests can fit into the space at this venue.

Bridezilla wants to be head of the largest table, which seats 36 guests. This is shown on the floor plan. Work out how many guests she can invite to her wedding by seeing how many will fit in the venue space. The table and chair arrangements must follow the sequence described on the previous page (page 12). So, each table arrangement will be a different size.

Hint: Try to get 5 more tables in this floor plan. Each table should seat fewer than 36 guests. There should be space between the chairs from all the tables so that guests do not bump against each other when getting up from the table.

36 guests

Number of guests: _____

Fabulous Fibonacci and the bunnies   solve
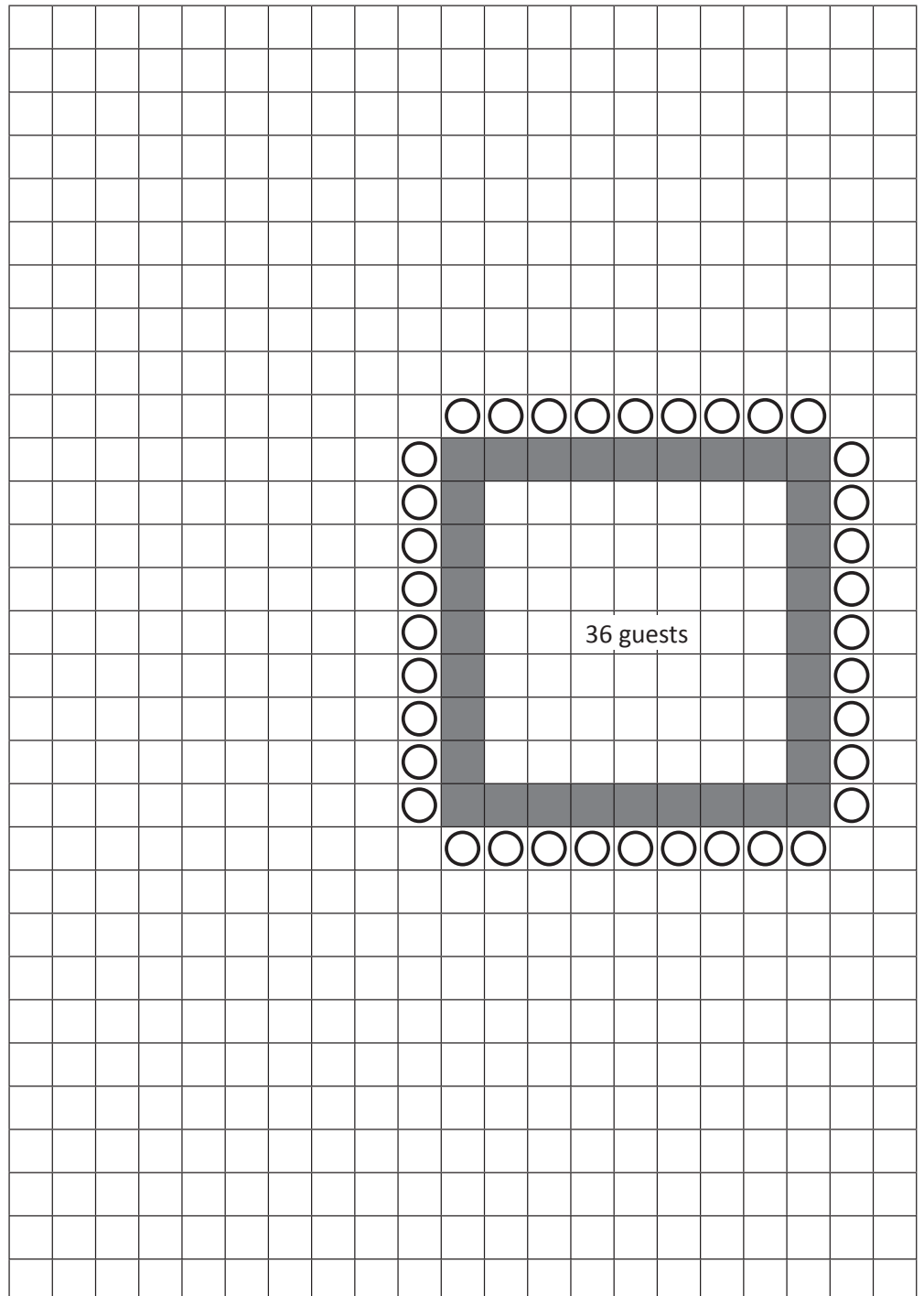
A famous mathematician by the name of Leonardi di Pisa became know as Fibonacci after the number sequence he discovered. He lived in 13th century Italy, about 200 years before another very famous Italian, Leonardo da Vinci.

His number sequence can be demonstrated by this maths problem about rabbits.

"How many pairs of rabbits will there be a year from now, if …?"

1 You begin with one male rabbit and one female rabbit. These rabbits have just been born.
2 After 1 month, the rabbits are ready to mate.
3 After another month, a pair of babies is born – one male and one female.
4 From now on, a female rabbit will give birth every month.
5 A female rabbit will always give birth to one male rabbit and one female rabbit.
6 Rabbits never die.

Month	Babies from 1st pair	Babies from 2nd pair	Babies from 3rd pair	Total pairs of rabbits
1				🐰
2				🐰
3	🐰			🐰🐰
4	🐰			🐰🐰🐰
5	🐰	🐰		🐰🐰🐰🐰🐰
6	🐰	🐰	🐰	🐰🐰🐰🐰🐰🐰🐰🐰

Key

🐰 = 1 pair of rabbits

What to do

Look carefully at the table above to understand the problem. If we kept going, the table would get very wide indeed and quite confusing! So it is up to you to figure out the pattern. Here is a closer look. Can you see what is happening? What are the next 2 numbers?

1	1	2	3	5	8		

Now, back to the bunnies. Use the table below to answer Mr Fibonacci.
"How many pairs of rabbits will there be a year from now?"

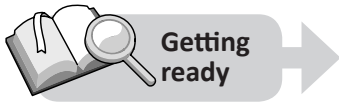| Months	1	2	3	4	5	6	7	8	9	10	11	12
Pairs of bunnies	1	1	2	3	5							

What to do next
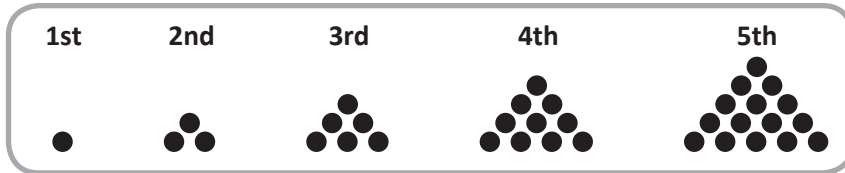
Fibonacci now wants to know:
"How many pairs of rabbits will there be 2 years from now?"

Use a calculator. *Hint: The table below should just continue from the previous one.*

Months	13	14	15	16	17	18	19	20	21	22	23	24
Pairs of bunnies												

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Patterns and Algebra

Triangular numbers   investigate

1st	2nd	3rd	4th	5th

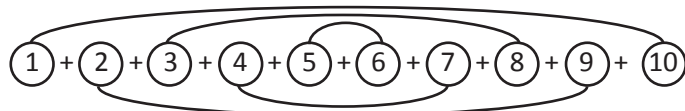
Write a number sentence for each part of the Triangular number
pattern and continue to complete this list:

1st   1   = 1

2nd   3   = 1 + 2

3rd   6   = 1 + 2 + 3

4th   10   = 1 + 2 + 3 + 4

5th   15   = _____

6th   21   = _____

7th   28   = _____

8th   ____   = _____

What to do

Let's investigate a faster way to find the 10th number:

$(1) + (2) + (3) + (4) + (5) + (6) + (7) + (8) + (9) + (10)$

Work from the outside in, until you reach the halfway adding the numbers.

What is the answer each time? _____

Half of 10 is _____ so that means we have 5 lots of 11, so the 10th triangular

number is _____.

What is the 20th number?

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20

Work from the outside in, until you reach the halfway adding the numbers.

What is the answer each time? _____

Half of 20 is _____ so that means we have _____ lots of _____,

so the 20th triangular number is _____.

What to do next

Find the 30th triangular number without writing down the numbers. _____

Hint questions:
What are the first and the last numbers? _____   *What do they add to?* _____

Getting ready

Pascal's triangle is named after Blaise Pascal and is fascinating to investigate because of all its hidden patterns. Blaise Pascal was born in France in 1623 and displayed a remarkable talent for maths at a very young age. His father, a tax collector, was having trouble keeping track of his tax collections, so he built his father a mechanical adding machine! (And you thought washing up after dinner was helpful!)

Pascal was actually lucky that this triangle was named after him as it was known about at least 5 centuries earlier in China.
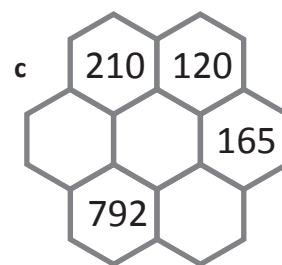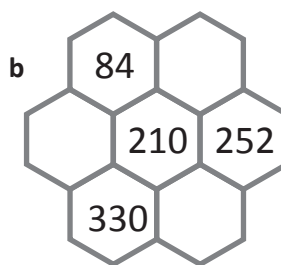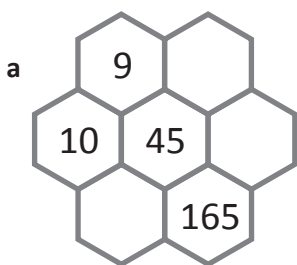
What to do

Look carefully at the numbers in the triangle. Can you see how you might go about completing it? Once you have worked this out, complete the rest of what you see of Pascal's triangle:
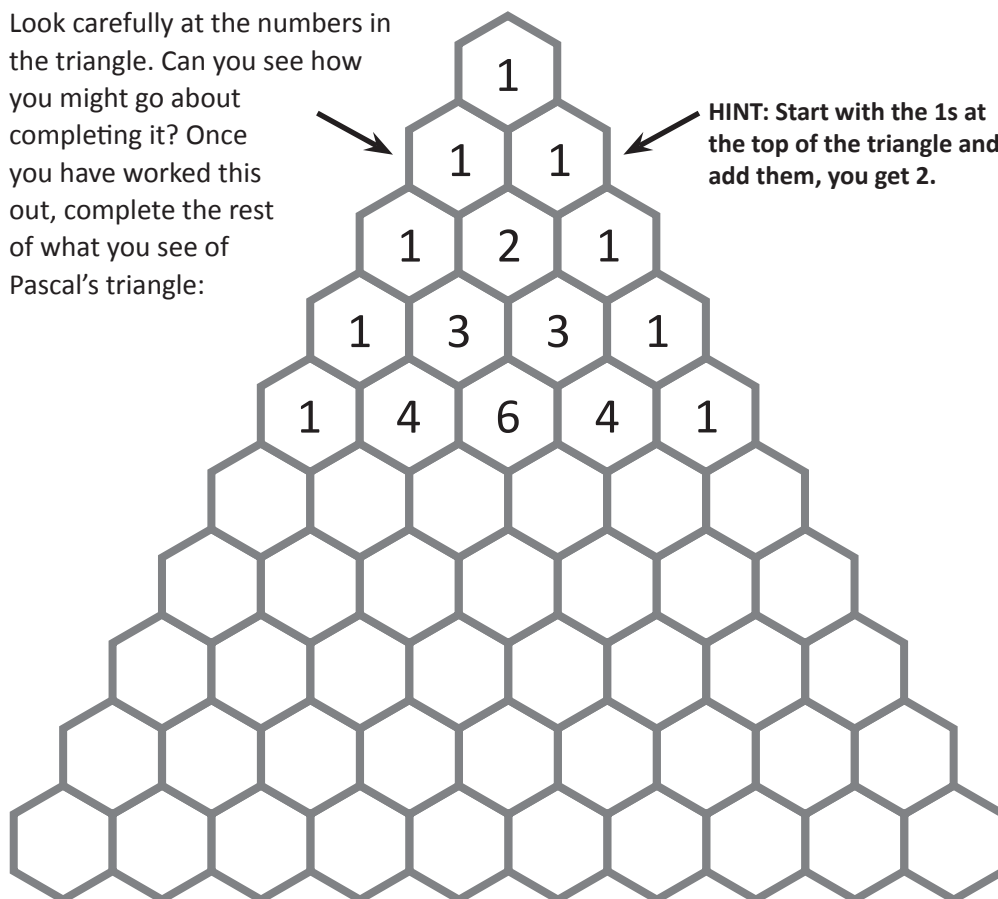
HINT: Start with the 1s at the top of the triangle and add them, you get 2.

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
```

Complete the missing sections of Pascal's triangle below.

a
```
    9
 10   45
      165
```

b
```
   84
      210   252
   330
```

c
```
  210  120
          165
      792
```

Patterns and Algebra

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What to do next

Check that the Pascal's triangle on page 16 is correct. Then copy the numbers into the triangle below and colour in all the multiples of 3 – red; hexagons with 1 less than a multiple of 3 – green; and all hexagons with 2 less than a multiple of 3 – blue.

```
              1
           1     1
        1     2     1
     1     3     3     1
  1     4     6     4     1
```
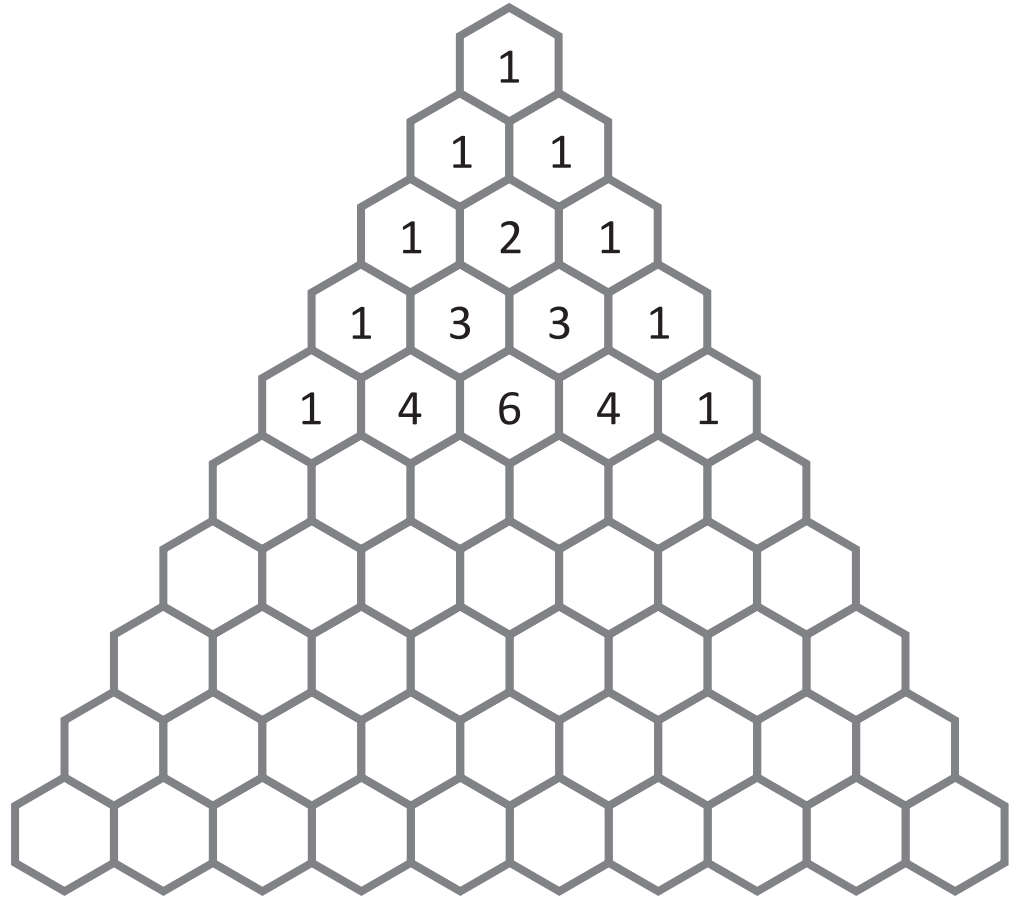
Can you see any other patterns in Pascal's triangle?

Look along the diagonals and describe as many patterns as you can.

See if you can find Fibonacci's sequence.

Algebraic thinking – making connections between unknown values

The balance strategy is what we use when we need to find the value of one symbol. Once we know the value of the first symbol, we can find out the value of the second symbol.

Clue 1 ☆ + 40 = 60

Clue 2 ☆ × △ = 100

> Doing the inverse to the other side of the equation cancels out a number and makes things easier to solve. This is called the balance strategy.

Use the balance strategy to find the value of ☆

☆ + 40 = 60

☆ + 40 = 60 − 40

☆ = 20

Now we know the value of ☆ we can work out the value of △

☆ × △ = 100

20 × △ = 100

20 × △ = 100 ÷ 20   Using the balance strategy we do the same to both sides which gives us the answer.

△ = 5

1   **Find out the value of both symbols:** ☆ △

a   Clue 1   ☆ − 15 = 45

    Clue 2   ☆ × △ = 120

☆ − 15 = 45

☆ − 15 = 45 + 15

☆ = ☐

☆ × △ = 120

☐ × △ = 120

△ = 120 ÷ 60

△ = ☐

b   Clue 1   ☆ × 9 = 81

    Clue 2   △ − ☆ = 96

☆ × 9 = 81

☆ × 9 = 81 ☐ ☐

☆ = ☐

△ − ☆ = 96

△ − ☐ = 96

△ = 96 ☐ ☐

△ = ☐

G SERIES   **2** TOPIC

Patterns and Algebra

Algebraic thinking – making connections between unknown values

2 Now that you have had practice following the clues and using the step prompts, try these on your own. Set your work out carefully and always use a pencil so that you can erase mistakes and try again.

a Clue 1 $\bigstar \times 8 = 64$

Clue 2 $\triangle - \bigstar = 75$

b Clue 1 $\bigstar \times 7 = 49$

Clue 2 $\bigstar + \triangle = 100$

3 Find out the value of both symbols: $\bigstar$ $\triangle$

a Clue 1 $6 \times \bigstar + 12 = 84$

Clue 2 $\bigstar \times \triangle = 96$

Steps for finding $\bigstar$

$6 \times \bigstar = 84 - \boxed{}$

$6 \times \bigstar = \boxed{}$

$\bigstar \times 6 = \boxed{}$

$\bigstar = \boxed{} \div \boxed{}$

$\bigstar = \boxed{}$

Now you can find $\triangle$

> It is easier if we put the star on the left hand side. We can swap numbers around with addition and multiplication.

REMEMBER

b Clue 1 $9 \times \bigstar - 42 = 21$

Clue 2 $\bigstar + \triangle = 100$

Steps for finding $\bigstar$

$\bigstar = \boxed{}$

Now you can find $\triangle$

Algebraic thinking – making connections between unknown values

4 This time you have 3 clues to work through. There are no step prompts, you are on your own except for one hint: start with the clue where you can find the value of one symbol. Set your working out clearly. Use each box to work out the value of each symbol.

> Find the value of these 3 symbols. You must look closely at each clue. There are hints along the way.

a

Clue 1 $2 \times$ ☆ $= 3 \times$ ◯

Clue 2 $3 \times$ ◯ $= 4 \times$ △

Clue 3 $6 \times$ ☆ $= 72$

☆ = ☐

◯ = ☐

△ = ☐

DISCOVER

Clue 1	Clue 2	Clue 3

b

Clue 1 $5 \times$ ☆ $= 3 \times$ ◯

Clue 2 $4 \times$ ◯ $= 60$

Clue 3 $45 \div$ ☆ $=$ △ $\div 4$

☆ = ☐

◯ = ☐

△ = ☐

Clue 1	Clue 2	Clue 3

Algebraic thinking – making connections between unknown values

If you were able to complete the last few pages, then you are ready for the next level of algebraic thinking. This time you have to work a bit harder to find the value of the first unknown. However it is easy if you follow these steps and look very closely at the clues. There are clues within the clues!

This page is a worked example. Each step is worked through to help you do this on your own on the next few pages.

Find the value of: ◯ △ ☆

> **Clue 1** ☆ + △ + ◯ + ◯ = 20
>
> **Clue 2** ☆ + △ = ◯ + ◯
>
> **Clue 3** ☆ = △ + 4

Clue 2 tells us that: ☆ + △ = ◯ + ◯

Looking at Clue 1, we can swap the star and triangle for 2 circles:

◯ + ◯ + ◯ + ◯ = 20   So, ◯ = 5

Clue 3 tells us that: ☆ = △ + 4

Looking at Clue 2, we can swap the star for the triangle plus 4, so:

△ + △ + 4 = ◯ + ◯

We know ◯ is 5, so: △ + △ + 4 = 10

Use the balance strategy   △ + △ + 4̸ = 10 − 4

△ + △ = 6

So, △ = 3

Now that we know the value of the triangle, we can find out the value of the star with Clue 3:

Clue 3   ☆ = △ + 4

☆ = 7

By looking closely at the clues, we have found out the value of all 3 symbols:

◯ = 5

△ = 3

☆ = 7

Algebraic thinking – making connections between unknown values

5 **This page is very similar to the last page. There are step prompts to help you along the way.**

Find the value of these 3 symbols: ◯ = ☐

You must look closely at each clue. △ = ☐

There are hints along the way. ☆ = ☐

Clue 1 ☆ + △ + ◯ = 50

Clue 2 ☆ + △ = ◯

Clue 3 ☆ = △ + 15

Clue 2 tells us that: ☆ + △ = ◯

Looking at Clue 1, we can swap the star and triangle for a circle. Now we have ◯ + ◯ = 50.

◯ + ◯ = ☐   So, ◯ = ☐

Clue 3 tells us that: ☆ = △ + 15

Looking at Clue 2, we can swap the star for triangle plus 15 so we have:

△ + △ + 15 = ◯

We know ◯ is _____ , so: △ + △ + 15 = ☐

Use the balance strategy △ + △ = ☐ ☐ ☐

△ + △ = ☐

So, △ = ☐

Now that we know the value of the triangle, we can find out the value of the star with Clue 3:

Clue 3   ☆ = △ + 15

☆ = ☐

G **2**

SERIES TOPIC

Patterns and Algebra

Algebraic thinking – making connections between unknown values

6 This time, there are 2 activities where you must use the clues. One has step prompts, the other does not:

a Find the value of these 2 symbols: ⬤ = [ ]

You must look closely at each clue. ☆ = [ ]

There are hints along the way.

Clue 1	☆ + ☆ = ⬤
Clue 2	☆ + ☆ + ⬤ = 200
Clue 3	☆ = ⬤ − 50

Clue 1 tells us that: [ ] + [ ] = [ ]

Looking at Clue 2, this means that: [ ] + [ ] = [ ]

So, ⬤ = [ ]

We know the value of ⬤, so we can put this into Clue 3:

Clue 3 [ ] = [ ] − [ ]

So, ☆ = [ ]

....

b Find the value of these 3 symbols: ⬤ = [ ]

You must look closely at each clue. △ = [ ]

☆ = [ ]

Clue 1	☆ + △ + ⬤ = 100
Clue 2	☆ + △ = ⬤
Clue 3	⬤ = △ + 40

Clue 1	Clue 2	Clue 3

Present puzzle   solve

Three pupils each brought in some presents for the Christmas Raffle.
Each pupil spent **£36.**

....

What to do Can you work out how much was spent on each present?
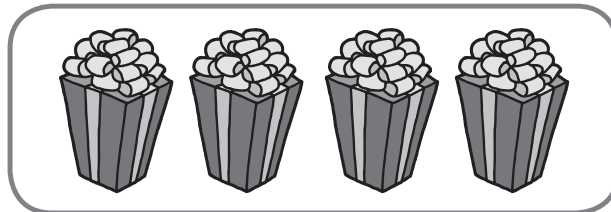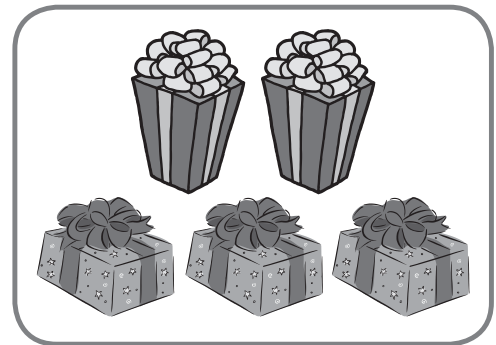
What to do next Label each present with the amount it is worth.

....

Patterns and Algebra

The lolly box   solve
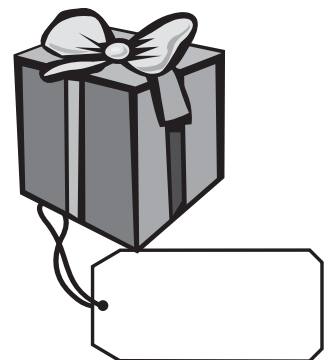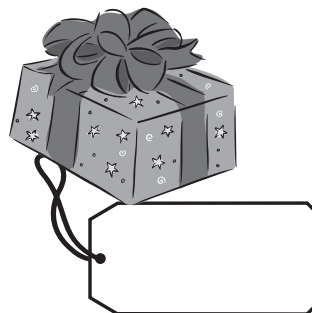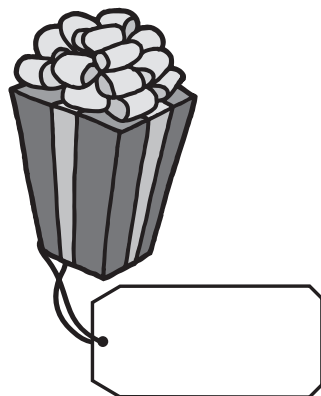
Getting ready

Miss Harley, the class teacher of 6H, enjoys getting her class to think mathematically by holding guessing competitions. Her most famous guessing competition was when she asked 6H to guess the number of cocoa puffs in a bowl if she used 250 mL of milk.

In her 2 latest competitions, she has said that the person who correctly guesses the exact contents of the box gets to take home all the lollies. This time she has given clues.
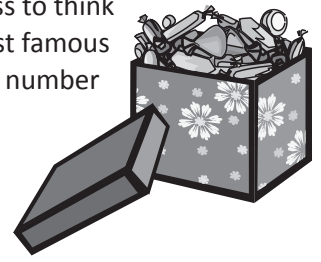
What to do

Read the clues for Competition 1 and look carefully at how to solve the problem. This will help you win Competition 2.

Competition 1 Clues

1 There are 36 lollies in the box which are a mixture of choc drops, mallow swirls and caramel dreams.
2 The number of mallow swirls equals four times the number of choc drops.
3 The number of caramel dreams is equal to the number of mallow swirls.

Choc drops		**Mallow swirls**		**Caramel dreams**
O	=	OO OO	=	OO OO
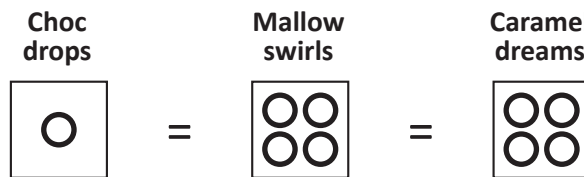
Look at the 3 types of lollies as 3 different groups. For every 1 choc drop, there are 4 mallow swirls and 4 caramel dreams.

So $9 \times \bigcirc = 36$

That means $\bigcirc = 4$

So there are:
4 choc drops
16 mallow swirls
16 caramel dreams

What to do next

Competition 2 Clues

See if you could win this box of lollies by using these 3 clues to work out the exact contents in the box. Follow the same steps as shown to you in Competition 1.

1 There are 84 lollies in the box which are a mixture of strawberry dreams, cola fizzes and lemon fizzes.
2 The number of cola fizzes equals twice the number of lemon fizzes.
3 The number of strawberry dreams equals double the number of cola fizzes.

Lemon fizzes		**Cola fizzes**		**Strawberry dreams**
	=		=	
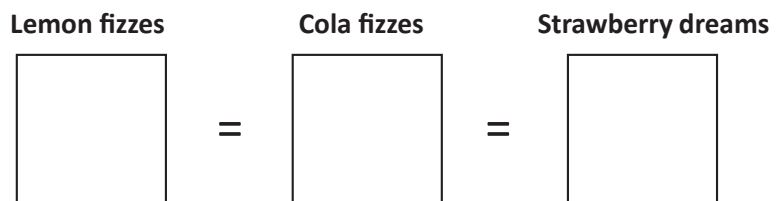
Solving equations – introducing pronumerals

Algebra normally uses letters of the alphabet to stand for unknown parts of an equation. These letters are known as pronumerals and are used in the same manner as symbols such as stars, triangles and boxes. Common letters used in algebra are: x, y, a, b, c, u and v.

$$\triangle - 12 = 38$$
$$\triangle - \cancel{12} = 38 + 12$$
$$\triangle = 38 + 12$$
$$\triangle = 50$$

Same equation

$$x - 12 = 38$$
$$x - \cancel{12} = 38 + 12$$
$$x = 38 + 12$$
$$x = 50$$

1 Use the balance strategy to find out the value of y:

a $y + 6 = 68$

b $y - 18 = 42$

c $y \times 8 = 72$

2 Using the balance strategy, solve each equation and then match the letters to the answers to solve this riddle: *What gets wetter and wetter the more that it dries?* The first one has been done for you:

| O | $x + 9 = 14$ |

$$x + \cancel{9} = 14 - 9$$
$$x = 14 - 9$$
$$x = 5$$

| E | $y - 5 = 29$ |

| A | $a + 7 = 15$ |

| W | $m + 5 = 19$ |

| T | $y + 8 = 25$ |

| L | $8 + x = 24$ |

8	17	5	14	34	16

G **3**

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Patterns and Algebra

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Solving equations – using pronumerals in an equation

In algebra, pronumerals are used to represent the unknown number or what we are trying to find out. Look at this example:

Amity's teacher gave the class a mystery number question:

"The sum of a mystery number and 18 is 36. What is the number?"

Amity used a pronumeral x to stand for the mystery number.

She wrote: $x + 18 = 36$

This is really saying, "mystery number plus 18 is 36."

Next, Amity used the balance strategy to solve the equation:

$$x + 18 = 36$$
$$x + 18 = 36 - 18$$
$$x = 18$$

1 **For each question, write an equation using the pronumeral x for the mystery number, then solve it.**

a The sum of 7 and a mystery number is 26.

b A mystery number increased by 15 is 48.

c A mystery number doubled is 64.

d The difference between a mystery number and 19 is 42.

Solving equations – using pronumerals in an equation

2 Find the value of x and y. First find the value of x by using the balance strategy, then you will be able to find the value of y. Show your working out:

a $x - 15 = 35$

$x \times y = 250$

$x = \boxed{}$ $y = \boxed{}$

b $x \times 9 = 72$

$x \times y = 48$

$x = \boxed{}$ $y = \boxed{}$

c $x \div 7 = 8$

$x + y = 60$

$x = \boxed{}$ $y = \boxed{}$

....

3 Find the length of the side of each of these shapes with algebra. Here you will be using pronumerals to represent the unknown number and the balance strategy to solve the equation. The first one has been done for you.

a If the perimeter of this square is 28 cm, find the length of one side. Call the side x.
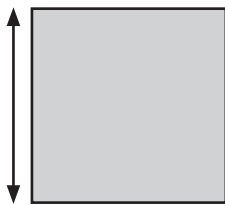
$$x \times 4 = 28$$
$$x \times \cancel{4} = 28 \div 4$$
$$x = 28 \div 4$$
$$x = 7 \text{ cm}$$

b The perimeter of this pentagon is 40 cm. Find the length of one side. Call the side y.

$$y \times 5 = 40$$

Patterns and Algebra

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Solving equations – using pronumerals in an equation

4 Algebra can help us find out the value of unknowns or mystery numbers. Look at how this perimeter riddle is solved and then solve the rest in the same way. Call each unknown y. **The first one has been done for you.**

a I am a length between 1 cm and 10 cm. When you add 4 cm to me you get the total length of one side of a square which has a perimeter of 28 cm. What am I?

$$(y + 4) \times 4 = 28$$
$$(y + 4) \times \cancel{4} = 28 \div 4$$
$$y + 4 = 7$$
$$y + \cancel{4} = 7 - 4$$
$$y = 3 \; cm$$

b I am a length between 1 cm and 10 cm. When you add 2 cm to me you get the total length of one side of an octagon which has a perimeter of 40 cm. What am I?
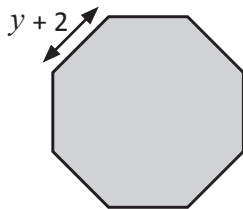
$$(y + 2) \times \boxed{} = \boxed{40}$$
$$(y + 2) \times \boxed{} = 40 \div \boxed{}$$
$$y + 2 = \boxed{} - \boxed{}$$
$$y = \boxed{} \; cm$$

c I am a length between 1 cm and 10 cm. When you add 5 cm to me you get the total length of one side of a pentagon which has a perimeter of 40 cm. What am I?
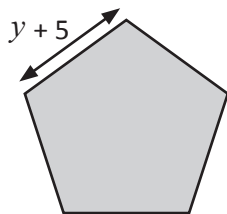
$$(y + 5) \times 5 = 40$$

....
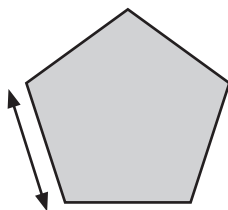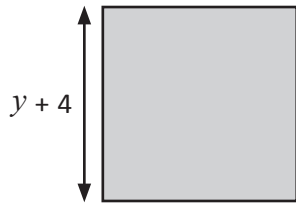
5 Read the mystery number riddle, then solve using algebra. Write the information as an equation, use y to stand for the unknown. Show all your working:

a I am thinking of a number between 1 and 10. When I add 2, then divide by 3 and multiply by 5, I get 10. What is the number?

$$(y + \boxed{}) \div \boxed{} \times \boxed{} = 10$$
$$(y + \boxed{}) \div \boxed{} \times \boxed{\diagup} = 10 \div 5$$
$$y + \boxed{} \div \boxed{\diagup} = \boxed{}\boxed{}\boxed{}$$
$$y + \boxed{\diagup} = \boxed{}\boxed{}\boxed{}$$
$$y = \boxed{}$$

b I am thinking of a number between 1 and 10. When I add 5, then divide by 4 and multiply by 5, I get 10. What is the number?

Solving equations – simplifying algebraic statements

An algebraic statement is part of an equation.

Sometimes algebraic statements can have the same variable many times.

To simplify a + a + a + a + a, we would rewrite it as 5a.

5a means 5 × a which is the same as a + a + a + a + a, but is much easier to work with.

1 **Match these algebraic statements by connecting them with a line:**

$k + k + k + 6$	$3k + 6$
$k + k + k + k$	$k + k + k + 2$
$3k + 2$	$4k$
$6k + k + k + 10$	$8k + 10$

2 **Simplify these statements. The first one has been done for you:**

a $3k + k + 4k =$ $8k$

b $6x + 2x + x =$ ☐

c $b + b + 5b =$ ☐

d $8y + 5y - y =$ ☐

3 **Complete the algebraic addition stacks. Here is a simple example to start you off. Blocks underneath must add to give the block above. In this example, $3a + 4a = 7a$.**

```
        7a
    3a      4a
```

a
```
        [   ]
   14b      [   ]
 5b     9b      7b
```

b
```
        22a
    [   ]     9a
 9a     4a      [   ]
```

c
```
        [   ]
   6x       13
 2x     [   ]     9x
```

d
```
        [   ]
   [   ]     25y
14y     7y      [   ]
```

G3
SERIES TOPIC

Patterns and Algebra
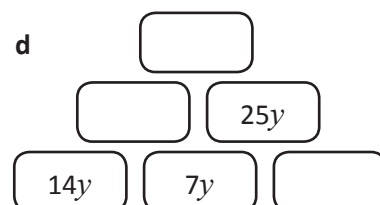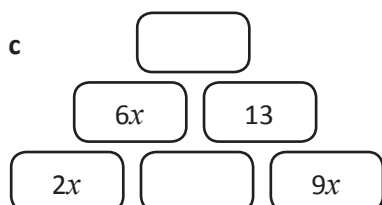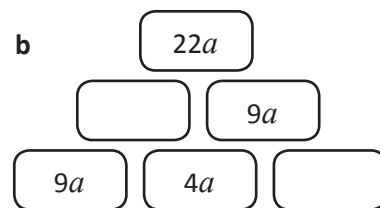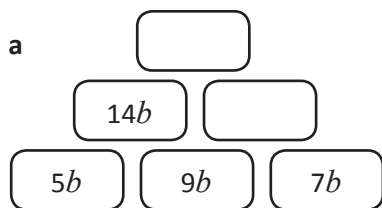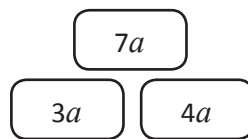
Solving equations – simplifying algebraic statements

Remember with algebraic statements, a letter next to a number just means multiply. $6y$ means $6 \times y$.
You can add and subtract pronumerals that are the same, just like you would regular numbers

$$5y + 9y = 14y \qquad 20a - 16a = 4a$$

4 **Use what you know about algebraic statements to solve these equations:**

a $2a + 3a = 15$

$5a \qquad = 15$ ← Use the balance strategy to find out what a is.
$5a$ means $5 \times a$, so with the balance strategy you must do the inverse which is ÷ 5 to both sides.

$5a \div 5 = 15 \div 5$

$a \qquad = 3$

b $9b - 5b = 24$

$\boxed{} = 24$

$\boxed{} \div \boxed{} = 24 \div \boxed{}$

$b = \boxed{}$

c $6c - 2c = 36$

$\boxed{} = 36$

$\boxed{} \div \boxed{} = 36 \div \boxed{}$

$c = \boxed{}$

5 **Solve this riddle in the same way as the questions above:**
What is as light as a feather but impossible to hold for long?

| H | $16r - 4r = 48$ |

| R | $9p - 2p = 35$ |

| A | $8i + 5i = 39$ |

| T | $10f - 3f = 42$ |

| B | $7m + 2m = 63$ |

| E | $7x - x = 54$ |

Your …

7	5	9	3	6	4

Happy birthday   solve

Getting ready

Three children are having a birthday party.

Can you work out how many candles need to go on each cake?

What to do

1 Read the clues.

2 Show your working.
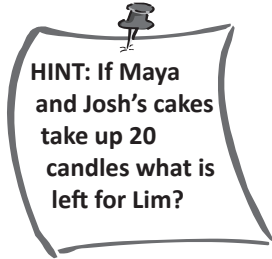
Clue 1 Maya and Josh have 20 candles.

HINT: If Maya and Josh's cakes take up 20 candles what is left for Lim?

Clue 2 Maya and Lim have 13 candles.

Clue 3 Lim and Josh have 15 candles.

Clue 4 There are 24 candles altogether.

What to do next

Draw the right amount of candles on each cake:
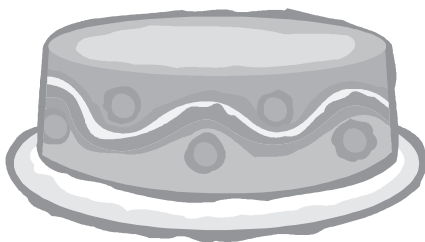
Happy Birthday Maya   Happy Birthday Lim   Happy Birthday Josh

Patterns and Algebra

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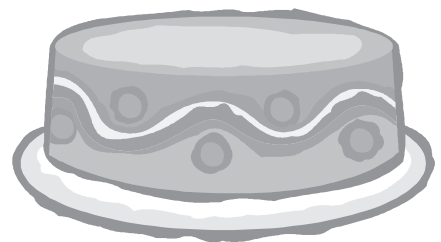
Squelch juiceteria
solve
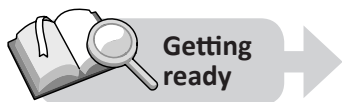
Getting ready

You work at Squelch Juiceteria, a popular juice bar serving delicious concoctions to go.

The manager has left you in charge of writing the daily specials on the board. She has texted you the four different juices she wants you to write up but forgot to text the prices and now her phone is turned off.

Squelch Juiceteria Specials

Mango Tango

Strawberry Squeeze

Cherry Bliss

Apple Berry

What to do

Use algebra with the clues below to work out the missing prices.

Look carefully at this first example and follow the steps to work out the rest.

Clue 1 A Mango Tango and a Strawberry Squeeze costs £9.

Clue 2 A Mango Tango costs £3 more than a Strawberry Squeeze.

Use algebra to find out the cost of each. Use m for Mango Tango and s for Strawberry Squeeze.

Step 1 *Write clues as algebra:*

$$m + s = £9$$
$$m - s = £3$$

Step 2 *Combine the clues into one statement to cancel out one unknown:*
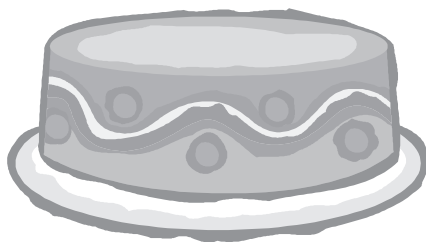
$$m + s + m - s = £9 + £3$$
$$m + s + m - s = £12$$

Step 3 *Work out cost of second juice:*

$$m + m = £12 \qquad m = \boxed{}$$
$$\boxed{} + s = £9 \qquad s = \boxed{}$$

Clue 1 A Cherry Bliss and an Apple Berry cost £12.

Clue 2 A Cherry Bliss costs £1 more than an Apple Berry.

Use algebra to find out the cost of each. Use c for Cherry Bliss and a for Apple Berry.

Step 1 *Write clues as algebra:*

$$c + a = \boxed{}$$
$$c - a = \boxed{}$$

Step 2 *Combine the clues into one statement to cancel out one unknown:*

Step 3 *Work out cost of second juice:*

Properties of arithmetic – order of operations

Mr Gain wrote this equation on the board: 4 + 5 × 7 = ?

Max performed the operation of addition first, then multiplication; Amity performed multiplication first, then addition. Now they are confused—they can't both be right!

We need a set of rules so that we can avoid this kind of confusion.

This is why for some number sentences we need to remember the 3 rules for the order of operations.

Rule 1 Solve brackets.

Rule 2 Multiplication and division before addition and subtraction.

Rule 3 Work from left to right.

By following the rules, we can see that Amity was right. Rule 2 says you should always multiply before you add.

4 + 5 × 7 = 63

4 + 5 × 7 = 39

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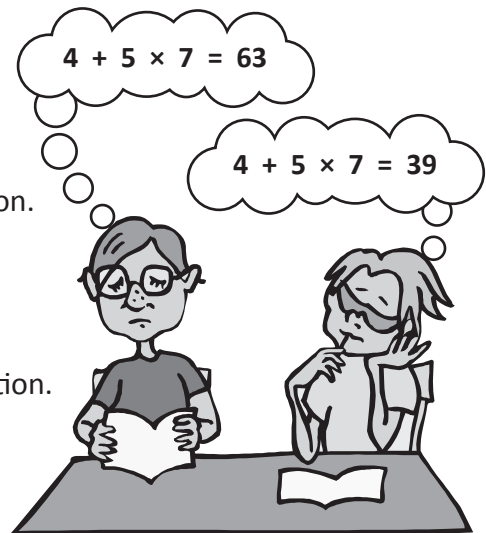
1 **Practise Rule 1, doing the brackets first:**

a 7 + (6 × 9)   =

b 8 + (4 × 7)   =

c 100 − (25 ÷ 5) =

d 30 ÷ (4 + 11) =

---

2 **Practise Rule 2, multiplication and division before addition and subtraction:**

a 100 − 4 × 8 =

b 60 + 25 ÷ 5 =

c 8 + 6 × 9   =

d 2 × 7 − 5   =

---

3 **Practise Rule 3, working from left to right:**

a 42 ÷ 6 × 4   =

b 46 + 10 − 20 =

c 32 + 4 − 16 =

d 72 ÷ 8 × 3   =

---

4 **Check the following sums based on what you know about the order of operations. Correct any that are wrong:**

a 50 − (45 ÷ 9) + 8 = 53

b 100 − (7 × 5) + (30 − 6) = 41

c (60 − 8) × 2 + (16 ÷ 4) − 32 = 140

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SERIES TOPIC

Patterns and Algebra

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Properties of arithmetic – order of operations

REMEMBER

5 Make these number sentences true by adding an operation (+, −, ×, ÷):

a 96 ☐ 3 ☐ 8 = 40

b 16 ☐ 4 ☐ 22 = 26

c 84 ☐ 12 ☐ 3 = 21

d 100 ☐ 5 ☐ 5 = 15

6 In each word problem there is an equation frame that solves each problem. Use it to solve the problem:

a | How much was the total bill if 5 people each had a sandwich worth £8 and 2 people had a drink for £3.

(☐ × ☐) + (☐ × ☐) = ☐

b | What is the total number of people at a party if 12 invitations were sent to couples, 7 people could not make it and 5 people turned up unannounced?

(☐ × ☐) − ☐ + ☐ = ☐

c | 30 children went to the water park. 12 went on the water slides first. The rest went in 3 equal groups to the swimming pool. How many were in one of the groups that went to the pool?

(30 − 12) ÷ ☐ = ☐

7 Work with a partner to see who can get the biggest number each round. Roll a die 3 times and write down the numbers in the equation frame. Compare your answers. The biggest answer wins ten points. The winner is the player with the highest score at the end.

Round 1: ☐ × (☐ + ☐) = ☐

Round 2: ☐ + ☐ × ☐ = ☐

Round 3: (☐ + ☐) × ☐ = ☐

My Score: / 30

Properties of arithmetic – commutative rule

Look at 13 + 16 + 4. The rules say we go from left to right but this sum is easier to answer if we add it like this: (16 + 4) + 13. The commutative rule lets us do this when it is all addition or all multiplication, no matter which order we do this the answer will be the same. But this is only if the sum is all addition or all multiplication. We can use brackets as a signal of what part of the sum to do first. Look at these examples:

7 + 34 + 23 = 64 is the same as (7 + 23) + 34 = 64

2 × 17 × 5 = 170 is the same as (2 × 5) × 17 = 170

Can you go both ways with subtraction and division?

THINK

1 Use brackets to show which pairs you should add first to make it easier:

a 17 + 3 + 8 = ☐   **b** 43 + 18 + 2 = ☐   **c** 62 + 5 + 15 = ☐   **d** 57 + 3 + 16 = ☐

2 Use brackets to show which pairs you should multiply first to make it easier:

a 7 × 25 × 4 = ☐   **b** 6 x 8 × 2 = ☐   **c** 50 × 4 × 3 = ☐   **d** 2 × 9 × 8 = ☐

3 Change the order and use brackets to make these equations easier:

Look for complements.

a 325 + 61 + 75 = ☐   ☐ + ☐ + ☐ = ☐

b 24 + 12 + 276 = ☐   ☐ + ☐ + ☐ = ☐
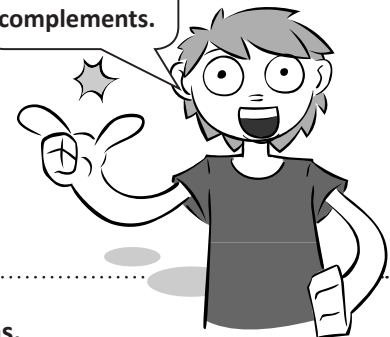
4 Using brackets and changing the order can make it easier to find unknowns. Look at the first question as an example, then try the rest.

REMEMBER

a

(△ × 12) × 5 = 120

△ × (12 × 5) = 120

△ × 60 = 120 ÷ 60

△ = 2

b

(△ + 36) + 14 = 100

c

40 + (160 + △) = 300

d

8 × (△ × 9) = 144

Patterns and Algebra

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Properties of arithmetic – commutative rule

5 Let's practise adding numbers in the order that makes it easier to add. Make a path through each number matrix so that the numbers add together to get to the shaded box. You can't go diagonally and not all of the numbers need to be used. Start at the bold number:

a

325	**75**	42
61	25	82
12	80	70
		250

b

50	150	42
30	120	75
12	180	25
		300

c

15	85	50
85	70	40
120	80	100
		400

....

6 Write equations for these word problems. Once you are sure of which operation to use, order the numbers in way that suits you.

a Adele loves reading books. One weekend she read 8 pages on Friday night, 17 pages on Saturday night and 12 pages on Sunday afternoon. How many pages did she read that weekend?

b Two classes competed to see who could raise the most money for charity over 3 days. 6H raised £85 on Monday, £38 on Tuesday, £15 on Wednesday. 6F raised £75 on Monday, £29 on Tuesday, £25 on Wednesday. How much did each class raise?

c Luke has been collecting aluminium cans for a sculpture he is making. He has been collecting 5 cans a week for the past 13 weeks but still needs double this amount. How many cans does he need in total?

Properties of arithmetic – distributive rule

The distributive rule says that you can split a multiplication into two smaller multiplications and add them.

$$53 \times 4$$
$$(50 + 3) \times 4$$
$$(50 \times 4) + (3 \times 4)$$
$$200 + 12 = 212$$

This comes in handy when the numbers are bigger than normal times tables questions.

1 **Fill in the missing numbers for the multiplications:**

a $64 \times 5 = \boxed{}$

$(60 + 4) \times 5$

$(\boxed{} \times 5) + (\boxed{} \times 5)$

$\boxed{} + \boxed{} = \boxed{}$

b $73 \times 5 = \boxed{}$

$(70 + 3) \times 5$

$(\boxed{} \times 5) + (\boxed{} \times 5)$

$\boxed{} + \boxed{} = \boxed{}$

c $56 \times 5 = \boxed{}$

$(\boxed{} + \boxed{6}) \times 5$

$(\boxed{} \times 5) + (\boxed{} \times 5)$

$\boxed{} + \boxed{30} = \boxed{}$

d $84 \times 6 = \boxed{}$

$(\boxed{} + \boxed{}) \times 6$

$(\boxed{} \times 6) + (\boxed{} \times 6)$

$\boxed{480} + \boxed{} = \boxed{}$

2 **Colour match each step of the distributive rule. For example, colour the equation frame labelled 1 yellow and look for all the parts that match this equation and colour them yellow too. Then match equation 2 and so on. By matching all 5 equations, you will have the order of the letters that spell the answer to the question below:**

1	$(30 + 8) \times 6$	$(20 \times 2) + (7 \times 2)$	$40 + 14 = 54$ A
2	$(20 + 7) \times 2$	$(30 \times 6) + (8 \times 6)$	$180 + 48 = 228$ N
3	$(10 + 9) \times 3$	$(10 \times 3) + (9 \times 3)$	$30 + 27 = 57$ I
4	$(40 + 4) \times 4$	$(70 \times 3) + (2 \times 3)$	$210 + 6 = 216$ S
5	$(70 + 2) \times 3$	$(40 \times 4) + (4 \times 4)$	$160 + 16 = 176$ L

What part of a human is in the Guinness Book of Records for reaching the length of 7.51 metres?

_____ _____ _____ _____ _____
  1   2   3   4   5

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SERIES TOPIC

Patterns and Algebra

Properties of arithmetic – distributive rule

3 Fill in the missing numbers for the divisions:

You can also use the distributive rule with division.

a 84 ÷ 4 = ☐

(80 + 4) ÷ 4

(☐ ÷ 4) + (☐ ÷ 4)

☐ + ☐ = ☐

b 108 ÷ 4 = ☐

(100 + 8) ÷ 4

(☐ ÷ 4) + (☐ ÷ 4)

☐ + ☐ = ☐

The distributive rule can help us find unknowns if we reverse the first 2 steps.

$$(8 × \blacklozenge) + (3 × \blacklozenge) = 88$$
$$(8 + 3) × \blacklozenge = 88$$
$$11 × \blacklozenge = 88$$
$$\blacklozenge = 8$$

Both 8 and 3 are to be multiplied by the diamond so we can rewrite this as shown in line 2. Then you can use the balance strategy twice to find the value of the diamond.

4 Use the distributive rule in reverse to solve this problem:

a Over the weekend Blake's dad made 5 batches of cup cakes on Saturday and 7 batches on Sunday. How many were in a batch if the total amount that he made was 180?

$$(5 × \triangle) + (7 × \triangle) = 180$$
$$(5 + 7) × \triangle = 180$$

b Jenna and Mel made up a game where if you score a goal you get a certain number of points. Jenna scored 6 goals and Mel scored 5 goals. How many points did they each get if the total number of points was 66?

$$(6 × \triangle) + (5 × \triangle) = 66$$

Getting ready

Practise what you have learned in this topic by playing equation pairs with a friend. You will need to copy both this page and page 41, then cut out the cards.

C copy

What to do

1 After shuffling the cards, place the 8 question cards and 8 answer cards face down in 2 separate arrays like this:

2 Player 1 selects one card from each set and if the question and answer match, then the player takes both cards and has another turn. If they don't match then Player 1 must return the cards to the same position and then it is Player 2's turn.

3 Continue until there are no cards left.

4 The player with the most pairs wins. Players go through the winner's pairs.

$7 \times (5 + 3)$	56
$42 \div 6 \times 4$	28
$27 + 11 - 8$	30

$(40 + 6) \times 5$

230

$(20 \times 7) + (20 \times 4)$

220

$60 + 25 \div 5$

65

$8 + 6 \times 9$

62

$50 - (45 \div 9) + 8$

53