One Two

Series G

Rich Learning Tasks

Dr. Marian Small

Problem Solving and Reasoning

One Two

Contents

Name:

Number

Planting Trees

Kyla can plant about 300 trees in an hour.
Mia can plant a single tree in about 13 seconds.
If they work together, how long would it take them to plant 100 trees?

Planting Trees

Reasoning behind the task

In order to solve this problem, students are likely to work with equivalent units of measure. Knowing that Kyla can plant 300 trees in an hour also tells you she can plant 5 trees in a minute or that in 12 seconds she can plant one tree. Knowing that Mia can plant one tree in about 13 seconds means she can plant 4 trees in 52 seconds, so about $4\frac{1}{2}$ trees in a minute or about 270 trees in an hour. Then students need to think about how knowing how many trees the two, together, can plant in a minute, or the number of seconds it takes each to plant a tree, helps them figure out how long it would take to plant 100 trees.

A number of the questions below focus on using the information given to estimate. Estimation should always be encouraged in ratio and rate problems.

Curriculum coverage

- Number
- Ratio and proportion
- Measurement
- Time conversions

Expectations

All	Most	Some
• Recognise the need for converting units of time. • Estimate that it would take about 10 minutes.	• Calculate that it would take about 10.5 minutes to plant the 100 trees. • Describe how they solved the problem. • Recognise that Kyla is faster than Mia.	• Explain how time conversions show Kyla is faster than Mia. • Explain why converting units of measure is useful for solving the problem.

Planting Trees

Key questions

- How long does it take Kyla to plant a tree? Is this information useful?
- How many trees can Mia plant in an hour? Is this information useful?
- Who is faster, Mia or Kyla? How do you know?
- How long does it take Kyla and Mia to plant ten trees?

Scaffolding learning

- Think about what units of measure you might use to solve this problem.
- Using ratio and proportion convert measurements of time from a larger unit to a smaller unit and vice versa.
- Find out how many trees Kyla can plant in one minute if she can plant 300 trees in an hour (5).
- Find out how trees Mia can plant in one minute if she can plant one in 13 seconds (4.5 approx).
- Use ratio and proportion to determine how long approximately it would take Kyla and Mia to plant 100 trees together ($9\frac{1}{2}$ tree: 1 minute; 100 trees: 10.5 mins).

Challenge

Choose different speeds for Kyla and Mia that would require them to need exactly 12 minutes to plant 100 trees.

Name:

Number

Original Numbers

30% of Number A is the same as 40% of Number B.
What could A and B be?
Think of lots of answers if you can.

Original Numbers

Reasoning behind the task

Rather than asking a student to simply calculate a percent, this problem is built on relationships. For example, we could have asked: If A is 50% of B, what percent of B is $\frac{1}{2}$ A? Instead, this problem is just slightly more complex.

Some students can solve the problem by just trying lots of numbers and looking for possibilities. For example, they might notice that 30% of 40 is the same as 40% of 30, or they might notice that 30% of 100 is the same as 40% of 75.

If the student correctly determines that B is 75% of A, then they should be able to find numerous solutions using their knowledge of equivalent fractions by multiplying A and B by the same amount.

Curriculum coverage

- Number
- Percentage relationships

Expectations

All	Most	Some
• Find at least one pair of numbers that work.	• Find more than one pair of numbers that work. • Explain their calculations and working out. • Predict that B must be less than A with reasoning.	• Find that, in general, B is 75% of A and explain why. • Predict and explain why B must be less than A. • Show an understanding that once they find numbers for A and B then they can find numerous pairs using their knowledge of equivalent fractions by multiplying/dividing A and B by the same amount.

Original Numbers

Key questions

- Which number, A or B, has to be greater? Why?
- How do you find 30% of a number?
- Could either number be the double of the other? Why or why not?
- How can you easily find another pair of numbers once you have a pair that works?
- What percent of Number B is 15% of Number A? Why?
- What percent of Number A is 10% of Number B? Why?

Scaffolding learning

- Know how to find a percentage of a number, e.g. 10% of 30.
- Look at the relationship between 30% of a number and 40% of a number. Think about possible quantities.
- Choose a number for A which can be easily divisible by 10, e.g. 120.
- Find 30% of this number, e.g. 30% of 120 = 36. Now think of this number as 40% of Number B. How will you work Number B?
- Look at the relationship between Number A and Number B. Can you use your knowledge of equivalent fractions to find other pairs of numbers which fit the statement?

Challenge

60% of Number A is the same as 50% of Number B. How are A and B related?

Name:

Fitting In

Number

Look at the fraction wall.

What fraction fits into another fraction about $2\frac{1}{2}$ times?

Look for lots of possibilities.

1																			
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Reasoning behind the task

Asking students to find a fraction that fits into another exactly $2\frac{1}{2}$ times is actually asking a division or multiplication question, i.e. If $a \div b = 2\frac{1}{2}$, what are a and b? or if $2\frac{1}{2} \times a = b$, what are a and b? But using the fraction wall makes the task accessible even to those students whose skills with multiplying and dividing fractions are either missing or minimal. Those students can count on visual cues to help them.

Some students might use equivalent fractions to determine solutions. For example, since $\frac{2}{10} = \frac{1}{5}$ and $\frac{3}{15} = \frac{1}{5}$, that means $\frac{1}{10}$ fits into $\frac{1}{5}$ twice and $\frac{1}{15}$ fits into $\frac{1}{5}$ three times, so perhaps $\frac{1}{12}$ is a reasonable solution.

Although some students might use the wall to notice that $\frac{1}{5}$ and $\frac{1}{2}$ work, that $\frac{1}{10}$ and $\frac{1}{4}$ work, that $\frac{1}{15}$ and $\frac{1}{6}$ work, and that $\frac{1}{20}$ and $\frac{1}{8}$ work, some students will generalise and realise that any fractions of the form $\frac{1}{5}n$ and $\frac{1}{2}n$ work, even though they do not see them on the wall. Other students will notice that if $\frac{1}{10}$ and $\frac{1}{4}$ work, so do $\frac{2}{10}$ and $\frac{2}{4}$, or $\frac{3}{10}$ and $\frac{3}{4}$, etc.

The use of the term "about" leaves opportunity for students to determine other solutions, e.g., $\frac{1}{9}$ and $\frac{1}{4}$, or $\frac{1}{20}$ and $\frac{1}{9}$.

Curriculum coverage

- Number
- Equivalent fractions
- Multiplying and dividing fractions

Expectations

All	Most	Some
• Identify at least one pair of fractions where the smaller fraction fits into the greater between $2\frac{1}{4}$ and $2\frac{3}{4}$ times.	• Identify more than one pair of fractions where the smaller fraction fits into the greater between $2\frac{1}{4}$ and $2\frac{3}{4}$ times. • Explain how they solved the problem. • Begin to show awareness that the numerator and denominator can be multiplied by the same amount to determine another solution.	• Identify pairs of fractions using equivalent fractions and/ or fraction operations to help solve the problem; understand that the numerator and denominator can be multiplied by the same amount to find another solution. • Clearly explain how they solved the problem.

Fitting In

Key questions

- Could one fraction be from the bottom and one from near the top of the fraction wall? Why or why not?
- How could using equivalent fractions help you find an answer?

Scaffolding learning

- Look at fraction relationships using the fraction wall and how fractions fit into one another.
- What fraction fits into $\frac{1}{3}$ twice? Three times? How might that help you find a solution?
- Create an algebraic equation to help solve the problem: $a \div b = 2\frac{1}{2}$, or, $2\frac{1}{2} \times a = b$

Challenge

What fraction fits into another fraction about 1 and $\frac{1}{3}$ times?

Name:

Measurement

Combined Shape

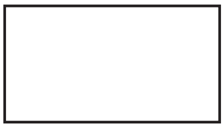
A shape made up of 2 trapeziums and a triangle has an area of 50 cm^2. Draw the shape, label the dimensions and the area of each sub-shape and prove that the total area is 50 cm^2.

How many different possibilities can you find?

Reasoning behind the task

This problem provides an opportunity to use the formula for finding the area of triangles, parallelograms, rectangles and trapeziums. Because they have the total area, instead of the area of each shape, students have much more scope for creating a solution.

Some students might start with a rectangle with an area of 50 cm^2, separate it into two trapeziums and a triangle and indicate the dimensions of the sub-shapes. They still need to describe the dimensions and areas of each sub-shapes.



Others will start with the trapeziums and triangle and use what they know about area formula to ensure that the total area is correct. Students also have an opportunity to reinforce their understanding of what different trapeziums can look like.

Providing a grid in the background allows students to count squares if they need to.

Curriculum coverage

- Measurement
- Area
- Area formula

Expectations

All	Most	Some
• Count grid squares to explore solutions and find the area of shapes.	• Create at least one shape made up of 2 trapeziums and a triangle with a total area of 50 cm^2. • Use a formula to determine the area of shapes.	• Create more than one shape made up of 2 trapeziums and a triangle with a total area of 50 cm^2. • Use a formula to determine the area of shapes. • Explain why there can be a number of solutions. • Realise there are many choices for the proportions of the three sub-shapes within the final shape.

Combined Shape

Key questions

- What can trapeziums look like?
- How do you find the area of a triangle?
- Do the trapeziums need to have the same area? Could they?
- Is it possible for all three shapes to have the same area? Explain.
- How do you know that there are many possible solutions?

Scaffolding learning

- Recap on the properties of a trapezium.
- Explore using the formula for finding the area of a rectangle to help you find the area of a triangle and a trapezium.
- Draw a rectangle and investigate separating it into two trapeziums and a triangle. Label your drawing with the dimensions of each sub-shape.
- Use the grid to allow you to count the squares should you need to.

Challenge

Create a shape made up of three or four of your own shape choices with a given total area.

Name:

Wrapping a Cuboid

The surface area of a cuboid is close to 75 cm^2.
What could the length, width, and height be?

Wrapping a Cuboid

Reasoning behind the task

There are different strategies students can use to find the surface area of a cuboid. Some might draw nets; others might visualise the faces in three dimensions. No matter how the problem is approached, students will need to recognise that cuboid have 3 pairs of congruent opposite rectangular faces.

The total area selected was 75 square units, rather than 70 square units, for example, because if students use whole-number dimensions the surface area is an even number, not an odd number. The use of 75 square units forces students to think about the phrase "close to." Some students, will use decimal or fractional dimensions.

Although some students will use the formula for the area of a rectangle, students who are uncomfortable with the formula can use the grid background to support them.

Curriculum coverage

- Measurement
- Surface area
- Nets

Expectations

All	Most	Some
• Use the grid to help them find the surface area • Identify a set of cuboid dimensions with a reasonable surface area.	• Identify a set of cuboid dimensions with a reasonable surface area and explore the effects of changing dimensions. • Use the formula for finding the area of a rectangle. • Understand why the surface area could not have been exactly 75 cm^2 if whole number dimensions were used.	• Identify a set of cuboid dimensions with a reasonable surface area and explain how changes in a dimension affect changes in surface area. • Explain how the surface area is found using formula. • Explain clearly why the surface area could not have been exactly 75 cm^2 if whole number dimensions were used.

Wrapping a Cuboid

Key questions

- How many separate areas do you have to calculate to figure out the surface area?
- Could the dimensions have been whole numbers or did they have to be fractions or decimals?
- If you increase the length by 1 and width by 1, do you keep the same surface area?
- If you interchange the length and the width, do you have the same cuboid with the same surface area or a different one?

Scaffolding learning

- Think about the properties of a cuboid; how many faces does it have?
- Consider why some of the faces have to have the same dimensions? Do all of them?
- Estimate possible surface areas for each congruent pair of faces using your knowledge of the total surface area of the cuboid.
- Use the grid to help you draw a net of a cuboid which has a surface area close to 75cm^2. Find the length, width and height of the cuboid.

Challenge

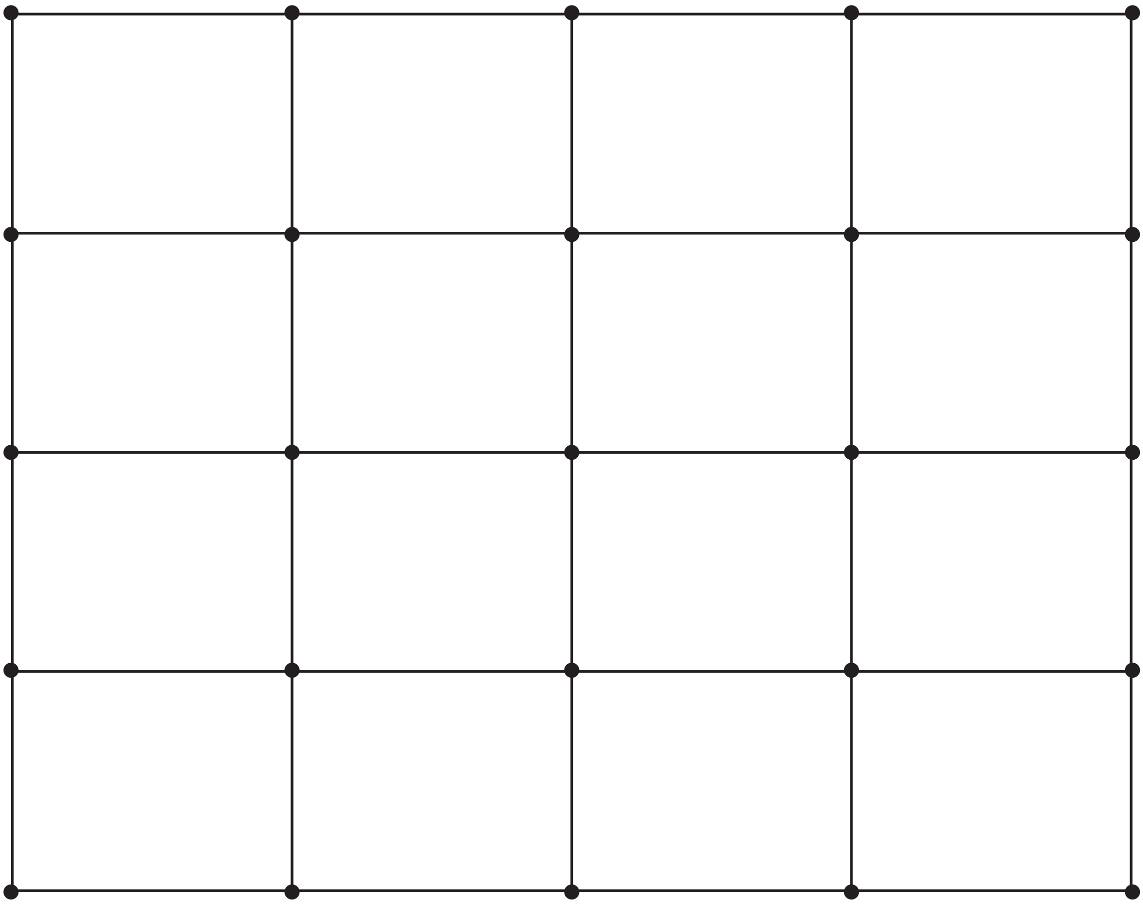
- Find the length, width and height of a cuboid with a surface area close to 30 cm^2.
- Find the dimensions of a triangular prism with a surface area close to 75cm^2.

Name:

Predicting Area

Make as many shapes as you can on the board where the vertices are positioned on the pegs, and so there is exactly one peg inside the shape.

How can you predict the area of the shape by knowing how many pegs are on its outside?

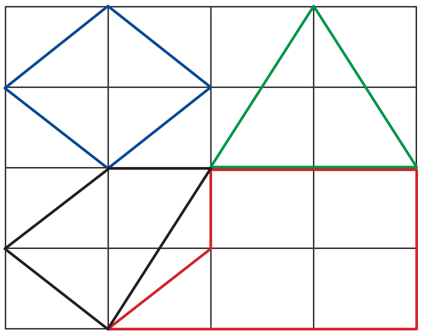


Predicting Area

Reasoning behind the task

Although most area formulas that students meet involve the use of lengths, widths, bases, or heights of shapes, there is an area formula related to shapes on grids that never mentions these sorts of dimensions. It is called Pick's (or Pic's) formula and it says that the area of a shape on a geoboard is calculated by dividing the number of pegs on the perimeter by 2, adding the number of inside pegs, and subtracting 1. Because the problem is set up requiring the number of inside pegs to be 1, the combined area of all of the shapes for this task will be half of the number of pegs on the perimeter.

There are several possible shapes, not just one.



Curriculum coverage

- Measurement
- Area

Expectations

All	Most	Some
• Create at least one shape which meets the required criteria.	• Create two or more shapes which meet the required criteria and begin to look for relationships between them. • Explore methods for finding the area for some or all of the shapes.	• Create enough shapes which meet the required criteria to notice that the area of the shape is the same as half the number of pegs on the boundary. • Predict the area of shapes accurately and use thoughtful strategies to find the area of the shapes they create. • Explain why very large shapes are not possible.

Predicting Area

Key questions

- Why can't your shape be too big?
- Could your shape be a rectangle with horizontal and vertical sides? Why or why not?
- How were the shapes you created alike? How were they different?
- Would a 5-sided shape be possible? If not, why not? If so, what would it look like?
- What did you notice about the number of pegs on the boundary of your shapes?
- What did you notice about all the areas?

Scaffolding learning

- Understand the terms 'vertices' and 'area'.
- Use grid paper, or peg boards, to explore shapes which meet the required criteria. Remember there should be exactly one peg inside the shape. How many possibilities can you find?
- Use the grid paper, or peg board, to help find the area of the shapes you have created by counting squares or parts of squares. Could you use the formula for area?
- Count how many pegs are on the outside of your shape. Investigate the relationship between this number and the total area of the shape.
- Research Pick's (or Pic's) formula *(the area of a shape on a geoboard is calculated by dividing the number of pegs on the perimeter by 2, adding the number of inside pegs, and subtracting 1)*. Apply this formula and compare with your original area calculations for each shape.

Challenge

Create shapes with exactly 2 pegs (or 3 pegs) inside and see how the area does or does not change from when there is 1 peg inside.

Name:

Equal for 10

$2x + 3$ is worth the same as another algebraic expression when $x = 10$ but not for other values of x.

What could the other expression be?

Are there other possibilities?

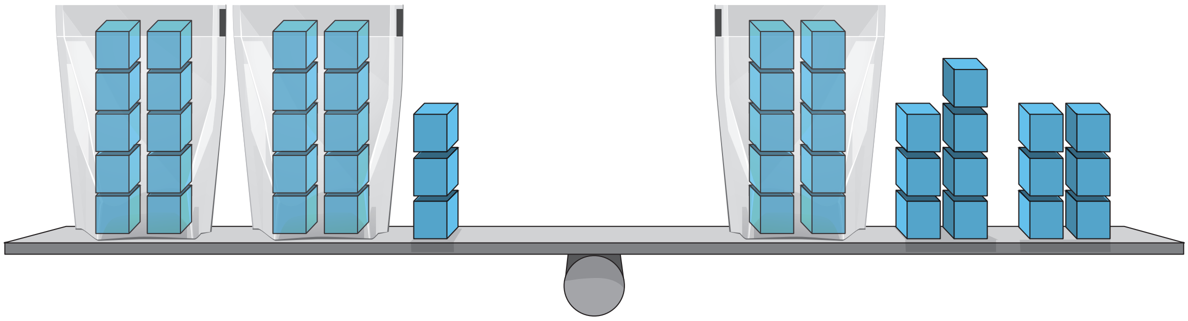
How could you use models to show that this is true?

Reasoning behind the task

It is essential for students' algebraic development that they realise that different expressions can be worth the same amount when they are evaluated.

Students might come up with expressions like $3x - 7$ or $x + 13$ or $\frac{x}{2} + 18$ which are worth the same as $2x + 3$ when $x = 10$, but not for other values of x. Other students might use an expression like $x + (x + 3)$ or $(4x + 6)/2$ which are other expressions for $2x + 3$ and are not only the same as $2x + 3$ when $x = 10$, but for all values of x.

The use of a model is to help students see that algebraic equalities can be modeled visually. For example, $2x + 3 = x + 13$ when $x = 10$ since the only way the sides below balance is if there are 10 cubes in each bag.



$2x + 3 = x + (x + 3)$ can be modelled as in table below, since the two expressions are the same.

x	x	3
x	x	3

Curriculum coverage

- Algebra

Expectations

All	Most	Some
• Create at least one expression equal to $2x + 3$ when $x = 10$.	• Create several expressions equal to $2x + 3$ when $x = 10$. • Explore using a model to show why the expressions are equal when $x = 10$. • Understand there can be even more possibilities.	• Create several expressions equal to $2x + 3$ when $x = 10$ and explain how they created them. • Effectively use a model to show why the expressions have to be equal when $x = 10$. • Explain why there have to be even more possibilities and why some are or are not equal to $2x + 3$ for other values of x.

Equal for 10

Key questions

- How did you find another expression for $2x + 3$?
- How does your model show that the expressions are equal when $x = 10$?
- Does your expression work when you apply other values for x? Why or why not?

Scaffolding learning

- Find the value of $2x + 3$ when $x = 10$.
- Investigate alternative means to getting 23 where $x = 10$, e.g. $2x + 3 = 3x - 7$. Record them as algebraic expressions.
- Identify a range of expressions for $2x + 3$ when $x = 10$. Test to see if the expression remains balanced if x is changed, e.g when $x = 5$. Can you find an expression which remains balanced no matter what x is?

Challenge

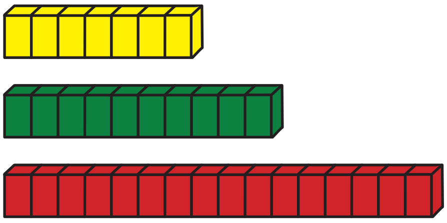
Explore how the choices of expressions that are equal to $2x + 3$ when $x = 10$, are alike and different from those that are equal to $2x + 3$ when $x = 20$.

Name:

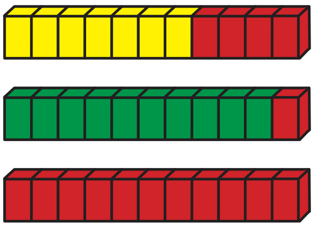
Building a Mean

One way to work out the mean (average) of a set of data is to use cubes to represent the data and move the cubes around to make the data equal. The mean is the length of the equal pieces of data.

For example, the mean (average) of 7, 10, and 16 is 11 because:



It turns into what you see below when you rearrange the cubes.



Your task is to find 6 numbers where the mean increases 4 of them but decreases 1 of them.

Use cubes to represent the data.

Try to find lots of possibilities.

Building a Mean

Reasoning behind the task

Students are introduced to how to use manipulatives to determine a mean in case this was new to them. But then they are asked to use what they see to solve a problem. One objective is for students to see that if 4 data values are increased and 1 is decreased, then the sixth data value must actually be the mean. Another objective is to realise that a fairly high value must be decreased if 4 values had to be increased. A third objective is to see that the total increase for the values that are increased matches the decrease for the value that is decreased. There are many possible solutions including, for example, 5, 5, 6, 7, 10, 27.

Curriculum coverage

- Patterns and algebra
- Calculating a mean using manipulatives

Expectations

All	Most	Some
• Identify at least one set of possible data values to meet the required criteria.	• Identify more than one set of possible data values to meet the required criteria. • Explain how they created the data set. • Recognise why there must be one large number; why one data value has to be the mean.	• Identify sets of possible data values to meet the required criteria and explain a strategy for creating many more solutions, e.g. adding the same amount to all numbers, or multiplying all numbers by the same amount. • Explain why there must be one large number; why one data value has to be the mean and why the total increase matches the single decrease.

Key questions

- Could you use all equal values? Why or why not?
- Could you start with the cubes instead of the numbers? How?
- Are there more low values or high values in your data? Why?
- What do you know about the value that was not increased or decreased?
- How did you find your six values?
- What do you notice about the total amount you increased the 4 numbers by compared to the amount you decreased one number by?
- How do you know that there are lots possible number combinations?

Scaffolding learning

- Find out what a 'mean' average is and how it can be useful in data handling.
- Know the term 'manipulatives' and use cubes to represent the data given in the task example to explore the process of using manipulatives to find the mean (average) of a set of data.
- Understand that the task requires you to find six numbers. Notice that four of these will be increased and one will be decreased to make the numbers balance. How many numbers does this leave unchanged? What does this tell us about the sixth number and the possibilities for the mean average?
- Think about your choice of six numbers. If one number needs to be decreased in order to increase four numbers then it will need to be a fairly high number.
- Draw a model to show your solution to the problem.

Challenge

Explore what the data set could be if three values were less than the mean and two values were greater than the mean.